# 1–4 Newton's Laws of Motion and Force Vectors

**Force:** Any push or pull.

**Newton's 1st Law:** Law of inertia, a restatement of Galileo's principle of inertia. Inertia is controlled by mass, the more mass the more inertia. Essentially an object at rest wants to stay at rest, while an object in motion wants to stay in motion.

**Newton's 2nd Law:** $\boxed{\sum F = ma}$. The Greek sigma proceeding $F$ is the mathematical notation for taking a sum. In reality there are countless forces interacting with every object in the known universe. When an object is standing still or moving at constant velocity all of these forces are counteracting and neutralizing each other. If a single additional force is applied (***applied force***) it can upset this balance forcing the object to change its inertia and accelerate. But, there may be more than one of these applied forces acting in a physics problem. These applied vector forces need to be added before a determination of the objects acceleration can be made.

**Net Force:** $\boxed{\sum F = ma}$

$\sum F_x$ is $\sum F$ in the ***x***-direction on the coordinate axis.

$\sum F_y$ is $\sum F$ in the ***y***-direction on the coordinate axis.

$\sum F_{\parallel}$ is the $\sum F$ parallel to a slope.

$\sum F_{\perp}$ is the $\sum F$ perpendicular to a slope.

**Newton's 3rd Law:** When two entities interact there is an equal and opposite force exerted on each object. Forces come in action-reaction pairs. For every ***action force*** there is an equal & opposite ***reaction force*** (***not*** an equal and opposite reaction). The reaction that is seen also depends on the mass of the object. If a high mass object encounters a low mass object, the one with less mass appears to be effected the most (either moves radically or sustains the most damage). This is due more to its low mass, since the force on both objects is the same.

**Common Forces:** In addition to the forces below, $F_{\text{any subscript that makes sense}}$, can be used.

There are many vectors in physics: displacement, velocity, acceleration, force, momentum, gravity fields, electric fields, magnetic fields, etc. The last three mentioned, gravity, electricity, and magnetism are field forces. These are normally associated with invisible force fields. When you push on a box the force is visible and obvious. Gravity pulling down on the box, toward earth, is not visible and not as obvious. These invisible forces are called field forces and motion is along the field lines. ***Often these field forces are not specifically mentioned, but they are implied by the conditions in the problem***.

| | | |
|---|---|---|
| $\sum F$ | $\sum F = ma$ | ***Sum of Force*** for linear motion, not used in circular motion. |
| $F_1$, $F_2$, … | | ***Applied Force***, some kind of push or pull. |
| $F_g$ or $W$ | $F_g = mg$ | ***Force of Gravity*** which is also known commonly as ***Weight***. |
| $T$ | | ***Tension*** is the force that acts along strings, ropes, chains, etc. |
| $N$ | $N = mg\cos\theta$ | ***Normal Force***: A contact force that exists when surfaces touch. It is always perpendicular to the surface. The angle $\theta$ refers to the angle of a sloping surface. On flat surfaces $\theta = 0^o$ and $\cos 0^o = 1$, so $N = mg$. However, this shortcut is only true as long as there are no applied forces or components of applied forces perpendicular to the surface. Additional $y$-forces or $y$-components of force can make the normal force one of the most difficult forces. See Ex. 4.4 on page 8. |
| $f$ or $F_{fr}$ | $f = \mu N$ | ***Force Friction***: A combination of the roughness of the surface, $\mu$, and the amount of force pushing the surfaces against each other (the normal force), $N$. |
| $F_c$ | $F_c = ma_c$ | ***Force Centripetal***: Sum of force for circular motion, not used in linear motion. |
| $F_E$ | $F_E = qE$ | ***Force of Electricity*** |
| $F_B$ | $F_B = qvB$<br>$F_B = BI\ell$ | ***Force of Magnetism*** on a charge moving in a magnetic field.<br>***Force of Magnetism*** on a current carrying wire in a magnetic field. |

***From a vector stand point, it does not matter what the exact force is when solving force problems. A force is a force is a force. The variable changes, $F_g$, $T$, $N$, $F_c$, $F_E$, $F_B$, or $F_1$, $F_2$, $F_3$, but it is still force. And when we finish with forces and move to momentum the letter will change to $p$ instead of $F$, but the vector problem solving technique remains the same.***

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**Free Body Diagram:** Imagine that you are looking down at a box from above, and that four people are pulling on ropes in the direction of the four arrows in Fig. 4.1. A diagram that just shows the object and the forces immediately acting on the object (and nothing else) is known as a free body diagram. As drawn the length of each force vector is an indication of its strength. Obviously persons 1 and 4 are stronger than 2 and 3. In free body diagrams the arrows do not have to be drawn proportionally, they just need to be pointing the correct way and be correctly labeled. The vectors are drawn coming out of the body. This allows an imaginary coordinate axis, shown in Fig. 4.1 by the dashed lines, to be superimposed through the center of the object. This is used to reference the direction of each force vector. Vector direction is simplified as + and − signs can be used to indicate direction along any axis. (In vector math the magnitudes +2 and –2 are equal. They are just opposite in direction.)

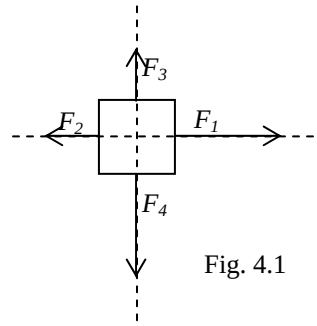

Fig. 4.1

## Hints to Complete Successful Free Body Diagrams

1. Identify any invisible force fields like gravity, electricity, and magnetism. Gravity is in every problem and points down.
2. If there is a string or rope, a force of tension exists along the string or rope and it is directed away from the object.
3. If there is a surface or contact point, there is a normal force directed away from and perpendicular to the surface.
4. Applied forces are the most obvious. They are in the text of the problem and may also be in any diagrams if provided.

## Finding the Force Resultant (Sum of Force) in Various Situations

### Example 4-1: All Vectors Lie Along an Axis.

Look at Fig. 4.1 and suppose that $F_1$ = 5 N, $F_2$ = 2 N, $F_3$ = 2 N, $F_4$ = 5 N.

1. Draw a Free Body Diagram (FBD), as shown in Fig. 4.1.
2. Write vector sum equation in any relevant direction(s). $\sum F_x = F_1 - F_2$ $\quad \sum F_y = F_3 - F_4$
3. Substitute known equations and/or values. $\sum F_x = 5 - 2 = 3$ $\quad \sum F_y = 2 - 5 = -3$
4. Solve. $\sum F_x = 3$ $\quad \sum F_y = -3$
5. If needed: Resolve components into a resultant. $\sum F = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{(3)^2 + (-3)^2} = \boxed{4.24N}$
6. If needed: Solve for the direction. $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{(-3)}{(3)} = \boxed{-45^o}$

### Example 4-2: Converting Vectors at Angles Into Components on the Coordinate Axis.

In Fig 4.2, suppose that $F_1$ = 5 N at 40°, $F_2$ = 2 N at 230°, $F_3$ = 2 N at 70°, $F_4$ = 5 N at –60°.

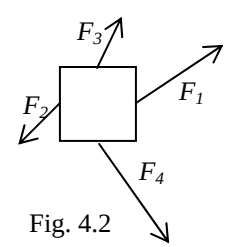

Fig. 4.2

1. Draw a FBD, as shown in Fig. 4.2.
2. State the vector sum equation in direction that matters
   $\sum F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$ $\quad \sum F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$
3. Substitute known equations and/or values
   $\sum F_x = F_1\cos\theta + F_2\cos\theta + F_3\cos\theta + F_4\cos\theta$
   $\sum F_y = F_1\sin\theta + F_2\sin\theta + F_3\sin\theta + F_4\sin\theta$
   $\sum F_x = 5\cos 40^o + 2\cos 230^o + 2\cos 70^o + 5\cos -60^o$ $\quad \sum F_y = 5\sin 40^o + 2\sin 230^o + 2\sin 70^o + 5\sin -60^o$
4. Solve $\sum F_x = 5.73N$ $\quad \sum F_y = -0.769N$
5. If needed: Solve for the total with Pythagorean theorem $\sum F = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{(5.73)^2 + (-0.769)^2} = \boxed{5.78N}$
6. If needed: Solve for the direction $\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{(-0.769)}{(5.73)} = \boxed{-7.64^o}$

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## Example 4-3: Using a Flexible Coordinate Axis System.

It will be easier in more complex problems if we set the direction that an object is moving to be positive. Once this direction is declared as positive all vectors pointing that way are positive, and those in the opposite direction are negative.

### *x*-direction:

If object is moving right then, direction of motion is right. Right vectors are positive.

$$\sum F_x = F_1 - F_2 - F_3$$

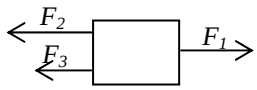


Fig 4.3a

If object is moving left, then direction of motion is left. Left vectors are positive.

$$\sum F_x = F_2 + F_3 - F_1$$

### *y*-direction:

If object is moving up, then direction of motion is up. Up vectors are positive.

$$\sum F_x = F_1 - F_2 - F_3$$

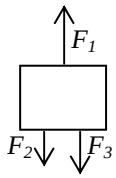


Fig 4.3b

If object is moving down, then direction of motion is down. Down vectors are positive.

$$\sum F_x = F_2 + F_3 - F_1$$

**Note:** The force vectors have already been assigned as positive and negative. So, ***plug in only positive numbers***, such as $g = +9.8$. ***Do not plug in negatives***, like, $-9.8$, as this will reverse the vector direction decided in the sum of force equation.

**Falling Bodies and Projectile Motion Revisited:** Try the above method for a falling body that is dropped at rest, or a projectile that is fired horizontally or downward. A vector is a vector, so whether it is force, velocity, or acceleration etc. use the same technique. A falling body is moving downward. So set the direction of motion as positive, thus down becomes positive. Any vector pointing down is now positive.

$y = \frac{1}{2}gt^2$ Both $g$ and $y$ point down and are positive. Everything is now positive. You are in charge of the problem. It does not matter what direction is positive. It matters that there is sign consistency. If down is defined to be positive then any vector pointing upward better have a positive. The beauty of using the direction of motion as positive and assigning plusses and minuses to the vector quantities is that you get to plug in positive values. ***(This is difficult to do in a falling body or projectile motion problem that travels in one direction(up) at the start of the problem and another direction (down) at the end of the problem. For these problems it may be easier to use the rigidly defined coordinate axis system with g as -9.8 that you have previously learned).***

## Example 4-4: Deciding On The Relevant Direction.

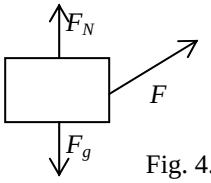


Fig. 4.4

Suppose a 10.0 kg box, in Fig. 4.4, is pulled along a flat surface by a 20.0 N force at a 30.0$^o$ angle with the horizontal. This problem is a little different than the previous ones. The *x*-direction and the *y*-direction are doing different things. The object is accelerating in the *x*-direction, while it is not moving in the *y*-direction. We can solve for the acceleration and for the normal force using the sum of forces in the relevant direction for each of these quantities.

1. Draw a FBD
2. Find the vector sum equation(s). This box will accelerate in the *x*-direction, so if acceleration data is requested, sum the forces in the *x*-direction. If the problem asks for a force in the *y*-direction, such as force normal, sum the forces in the *y*-direction.

$$\sum F_x = F_x \qquad \sum F_y = N + F_y - F_g$$

3. ***<u>Ask yourself what the object is doing in the relevant direction.</u>*** There are three choices:
   1. Standing still, $a = 0$. 2. Constant velocity, $a = 0$. 3. Accelerating, $a = a$.
   This problem is accelerating in the *x*-direction, while standing still in the *y*-direction.
4. Substitute known equations and values. *The known equations come from the table on page 6.*

$$ma_x = F\cos\theta \qquad ma_y = N + F\sin\theta - mg$$

$$10.0a_x = 20.0\cos 30^o \qquad 10.0(0) = N + 10.0\sin 30^o - (10.0)(9.8)$$

5. Solve

$$a_x = \frac{20.0\cos 30^o}{10.0} \qquad N = (10.0)(9.8) - 10.0\sin 30^o$$

$$a_x = \boxed{1.73\,m/s^2} \qquad N = \boxed{93.0N}$$

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**What Is The Object Doing?** It is critical to know how the object is moving.

1. Inertia: The object is following the principle of inertia. It lacks acceleration.
   a. Constant Velocity: $a = 0$, which means that $\sum F = ma = 0$.
   b. Standing Still: $a = 0$, $\sum F = ma = 0$. Standing still, $v = 0$, is just a constant velocity that happens to be zero.
2. Accelerating: $\sum F = ma$. $a$ is solved using this equation, or by other means and then is plugged into this equation.

## Example Scenario:

1. Standing still in a car: $\Delta v = 0$ $\quad a = 0$ $\quad \sum F = ma = 0$ Inertia.
2. Step on gas peddle: $\Delta v = +$ $\quad a = +$ $\quad \sum F = F_{engine} = ma = +$ Accelerates.

   Initially there is only one applied force acting in this problem, and it is the cars engine (really it is friction between tires and road). This force is in the direction of motion and is a positive force causing positive acceleration. But as soon as the car starts moving it experiences air resistance. It will continue to accelerate as long as the sum of force is positive.

   $\Delta v = +$ $\quad a = +$ $\quad \sum F = F_{engine} - F_{air} = ma = +$ Accelerates.
3. Reach cruising speed: $\Delta v = 0$ $\quad a = 0$ $\quad \sum F = F_{engine} - F_{air} = ma = 0$ Inertia.

   Why did it reach a constant velocity with the gas peddle still pressed? The car encounters air resistance, a second force. Air resistance is in the opposite direction and is negative. At first the air resistance is small, but grows until it equals the force of the engine. Adding the positive force of the engine to the now equal negative force of air resistance results in a $\sum F = ma = 0$, and inertia / constant velocity takes over. Note: equal and opposite force does not mean that the car stops. It continues doing whatever it was last doing, i.e., it moves with the final velocity from the acceleration phase.
4. Step on the brake: $\Delta v = -$ $\quad a = -$ $\quad \sum F = -F_{brake} - F_{air} = -ma = -$ Decelerates.

   The brakes (again, really friction) apply a force that opposes the direction of motion. Now there are two forces slowing the car. These negative forces decelerate the car.
5. Stopped: $\Delta v = 0$ $\quad a = 0$ $\quad \sum F = ma = 0$ Inertia.

### Example 4-5: Balanced Forces

***The forces parallel to the relevant direction add to a sum of force of zero.***

As an example: All the forces in the Fig. 4.5 are parallel to the $x$-axis.

Fig. 4.5

The sum of force is: $\sum F = F_1 + F_2 - F_3 - F_4$

$$\sum F = 3 + 2 - 4 - 1 = 0$$

The object does not accelerate, but rather has constant velocity (which includes $v = 0$).

***<u>This implies that constant velocity and standing still are the result of balanced forces, and this can be used as a shortcut.</u>***

**What if the problem were worded differently?**

The box in Fig. 4.6 is moving at constant velocity. What is $F_4$?

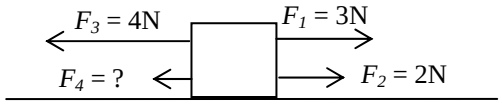

Fig. 4.6

Constant velocity implies that $\sum F = ma = 0$.

Now we know that this implies a balanced force scenario as well.

Solve using the ***sum of forces approach***.

$$\sum F = F_1 + F_2 - F_3 - F_4$$
$$0 = F_1 + F_2 - F_3 - F_4$$
$$F_1 + F_2 = F_3 + F_4$$
$$F_4 = F_1 + F_2 - F_3$$
$$F_4 = 3 + 2 - 4$$
$$F_4 = 1$$

Solving using ***balance force approach***.

All the forces in one direction (added together) must equal all the forces in the opposite direction (added together).

$$F_1 + F_2 = F_3 + F_4$$
$$F_4 = F_1 + F_2 - F_3$$
$$F_4 = 3 + 2 - 4$$
$$F_4 = 1$$

The balanced force approach is faster, as it eliminates the first two steps. ***<u>However, when an object is accelerating balanced forces do not apply, and the problem must start with a sum of forces equation.</u>***

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