1–8 Linear Momentum and Collisions

Momentum: $\boxed{\mathbf{p} = m\mathbf{v}}$ Quantity of motion (inertia in motion). Measure of how difficult it is to stop an object.

Impulse: $\boxed{\mathbf{J} = \mathbf{F}\Delta t = \Delta\mathbf{p}}$ Trade off between time taken to stop and force needed to stop. Velocity, acceleration, and momentum were understood early on in the developing days of physics. These concepts were used by Newton to establish his 2nd Law: $\mathbf{F}\Delta t = \Delta\mathbf{p}$ $\quad \mathbf{F}\Delta t = m\Delta\mathbf{v}$ $\quad \mathbf{F} = m\dfrac{\Delta\mathbf{v}}{\Delta t}$ $\quad \mathbf{F} = m\mathbf{a}$

Calculus: Force is the derivative of momentum $\boxed{\mathbf{F} = \dfrac{d\mathbf{p}}{dt}}$. Analogous to acceleration being the derivative of velocity $\boxed{\mathbf{a} = \dfrac{d\mathbf{v}}{dt}}$. If you divide the force and momentum vectors by the scalar mass you get the acceleration and velocity vectors. Physics is full of meaningful patterns. Impulse is the integral of force during a time interval $\boxed{\mathbf{J} = \int \mathbf{F}\ dt = \Delta\mathbf{p}}$. This means that if you take the derivative of impulse with respect to time you will calculate force $\boxed{\mathbf{F} = \dfrac{d\mathbf{J}}{dt}}$.

Collisions and Conservation of Momentum: Momentum in any situation must ***always*** be conserved. When two objects, each having momentum, collide the total momentum during the problem remains the same. If mass 1 and mass 2 are moving then they both a momentum. Their momentums add together to calculate a total momentum $\boxed{p_{total} = m_1 v_{1i} + m_2 v_{2i}}$. If they collide they might bounce off each other with different velocities than before, but the total momentum must remain the same $\boxed{p_{total} = m_1 v_{1f} + m_2 v_{2f}}$. If the initial momentums and final momentums are both equal to the same total momentum then the sum of the initial momentums must equal the sum of the final momentums $\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$. If there are more than two masses involved, just keep adding them to both sides.

Momentum is a Vector: Unlike energy (a scalar) which simply adds, momentum is a vector. Vectors have direction, and this means that the vector directions (like force vectors) must be accounted for in the math. You must decide on a positive direction in a conservation of momentum problem. Once decided any mass traveling in that direction has a positive momentum. Any mass traveling opposite the chosen direction has a negative momentum. Any mass traveling at an angle to the chosen direction must be split into components, with the x and y directions analyzed separately. Use the same strategies that were learned in forces.

Elastic Collision: Collisions in which kinetic energy is conserved. This can only happen when two objects do not touch each other. One example that may make sense and will be used latter in electricity is the collision between two protons. Protons have positive charges and in electricity like charges repel. If two protons approach each other head on the repulsion for each other will slow them to a stop before they touch one another. Then the repulsion will repel them away from each other. In effect they bounce off each other without touching

Inelastic Collisions: Collisions in which kinetic energy is lost. Since energy is never really lost, it must go somewhere. Lost energy is just energy that was lost by the system to the environment or it is energy that changed into a form that is not very recognizable. When masses collide and touch each other the masses vibrate. This vibration is heat. In collisions where objects touch each other some of the original kinetic energy is lost as heat.

Perfectly Inelastic Collision: The objects collide and stick together (one mass, one velocity) $\boxed{m_1 v_{1i} + m_2 v_{2i} = v_f\left(m_1 + m_2\right)}$.

Explosion: The opposite of a perfectly inelastic collision. A single object fractures and sends fragments in many directions. If we look at the simpliest case where it fractures into two bodies moving in opposite directions then $v_i\left(m_1 + m_2\right) = m_1 v_{1f} + m_2 v_{2f}$. Usually the original object is considered stationary in beginners examples, but it does not have to be $0 = m_1 v_{1f} + m_2 v_{2f}$. So this means that $\boxed{m_1 v_{1f} = -m_2 v_{2f}}$. The negative sign implies that one object must move in the opposite direction of the other.

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Example 8-1: Conservation of Linear Momentum

Elastic Collision: Mass 1, m_1 = 2.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 4.00 m/s. The collision is diagrammed before and after in Fig 8.1a. What is the velocity of mass 1 after the collision?

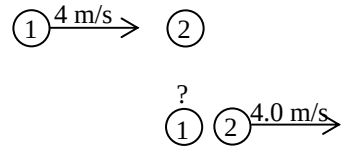

Fig 8.1a

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(2)(4) + (2)(0) = (2) v_{1f} + (2)(4) \qquad \boxed{v_{1f} = 0 m/s}$$

All the momentum from mass 1 was transferred to mass 2.

Inelastic Collision: Mass 1, m_1 = 4.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision mass 2 moves to the right at 5.30 m/s. The collision is diagrammed before and after in Fig 8.1b. What is the velocity of mass 1 after the collision?

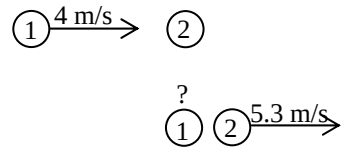

Fig 8.1b

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$(4)(4.00) + (2)(0) = (4) v_{1f} + (1)(5.30) \qquad \boxed{v_{1f} = 1.35 m/s}$$

Perfectly Inelastic Collision: Mass 1, m_1 = 4.00 kg, is moving at 4.00 m/s to the right. Mass 2, m_2 = 2.00 kg, is stationary and is hit by mass 1. After the collision the mass stick together. The collision is diagrammed before and after in Fig 8.1c. What is the velocity of the combined mass after the collision?

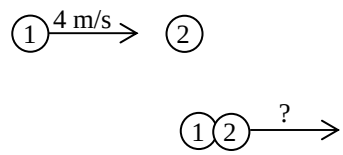

Fig 8.1c

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$(4)(4.00) + (2)(0) = (4+2) v_f \qquad \boxed{v_{1f} = 2.67 m/s}$$

Explosion: A large mass fractures into two smaller masses, m_1 = 4.00 kg and m_2 = 2.00 kg. Mass 2 moves to the right at 2.0 m/s. How fast is mass 1 going after the explosion?

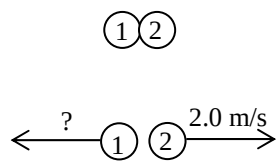

Fig 8.1d

$$(m_1 + m_2) v_i = m_1 v_{1f} + m_2 v_{2f}$$

$$(4+2)(0) = (4) v_{1f} + (2)(2) \qquad \boxed{v_{1f} = -1 m/s}$$

<u>The minus sign in the last example means that the mass is going to the left. Remember to watch the minus signs. Harder problems will have masses moving in different directions. Missing the sign convention will destroy the problem.</u>

Collisions and Energy: **Total energy is always conserved**, $E_{1i} + E_{2i} = E_{1f} + E_{2f}$.

However, kinetic energy is only one form of energy. If we look only at kinetic energy it sometime seems to disappear.

Kinetic energy is conserved only in <u>elastic collisions</u>. $K_{1i} + K_{2i} = K_{1f} + K_{2f}$

Kinetic energy is lost (dissipated) in <u>inelastic collisions</u>. $(K_{1i} + K_{2i}) - K_{lost} = (K_{1f} + K_{2f})$

But where does the lost kinetic energy go? When two or more bodies collide the molecules that they are composed of are set into faster vibration. The speed of molecules is directly proportional to temperature, and temperature is one component of ***<u>internal energy</u>***. So while kinetic energy is lost, total energy is not. The kinetic energy lost is turning into another form of energy, which is not one of the ***<u>mechanical energies</u>*** (kinetic and potential). It is now in the form of vibrating atoms and molecules: ***<u>internal energy</u>*** (a thermal energy).

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Example 8-2: Two dimensional Collision

Mass 1, m_1 = 2kg, is moving at 4m/s to the right. Mass 2, m_2 = 1kg, is stationary and is hit by mass 1 just a little off center. This causes mass 2 to move a 3m/s at an angle of 20° below the x-axis. The collision is diagrammed in Fig 8.2a (initial conditions) and in Fig. 8.2b (final conditions). What is the speed and direction of mass 1 after the collision?

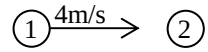

Fig 8.2a

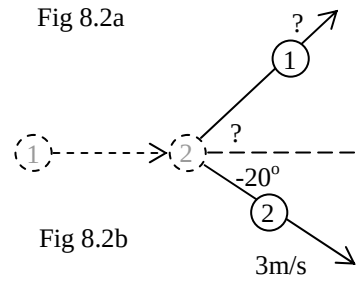

Fig 8.2b

The total momentum before the collision must equal the total momentum after. But after the collision the masses are moving at angles to the initial motion of m_1. We have to work in components. The total momentum in the x-direction must be conserved and the total momentum in the y-direction must be conserved.

***x*-direction**: Two bodies before and two after.

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$(2)(4) + (1)(0) = (2)v_{1fx} + (1)\left(3\cos\left(-20^o\right)\right) \qquad v_{1fx} = 2.59 m/s$$

***y*-direction**: This dimension is an explosion. Initially there is no motion in the y-direction at all. Then there are two bodies moving in opposite directions.

$$(m_1 + m_2)v_{iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

$$0 = (2)v_{1fx} + (1)\left(3\sin\left(-20^o\right)\right) \qquad v_{1fy} = 0.513 m/s$$

When you have two component vectors Pythagorean Theorem them back together and use arctangent to find the angle.

$$v_f = \sqrt{v_{1fx}^{\ 2} + v_{1fy}^{\ 2}} = \sqrt{(2.59)^2 + (0.513)^2} = \boxed{2.64 m/s} \qquad \theta = \tan^{-1}\frac{v_{1fy}}{v_{1fx}} = \tan^{-1}\frac{(0.513)}{(2.59)} = \boxed{11.2^0}$$

What amount of kinetic energy is lost? Remember energy is a directionless scalar, so x and y are meaningless.

$$(K_{1i} + K_{2i}) - K_{lost} = (K_{1f} + K_{2f})$$

$$\left(\frac{1}{2}mv_{1i}^{\ 2} + \frac{1}{2}mv_{2i}^{\ 2}\right) - K_{lost} = \left(\frac{1}{2}mv_{1f}^{\ 2} + \frac{1}{2}mv_{2f}^{\ 2}\right)$$

$$\left(\frac{1}{2}(2)(4)^2 + \frac{1}{2}(1)(0)^2\right) - K_{lost} = \left(\frac{1}{2}(2)(2.64)^2 + \frac{1}{2}(1)(3)^2\right) \qquad \boxed{K_{lost} = 0.0304 J}$$

Example 8-3: Ballistic Pendulum

The ballistic pendulum is used to determine projectile speed. The sequence of events is as follows. First a projectile, like a bullet (b), is fired into a block (B). This collision is perfectly inelastic, so $\boxed{m_b v_{bi} + m_B v_{Bi} = (m_b + m_B)v_f}$ is used to solve for the v_f of the bullet block combination. v_f for this first phase becomes the v_0 for the second phase. In the second phase the bullet block combination swings as a pendulum to a new height, as shown in the diagram to the right. Conservation of energy applies. $\boxed{\frac{1}{2}mv_0^{\ 2} = mgh}$ is used to determine the height of the swing.

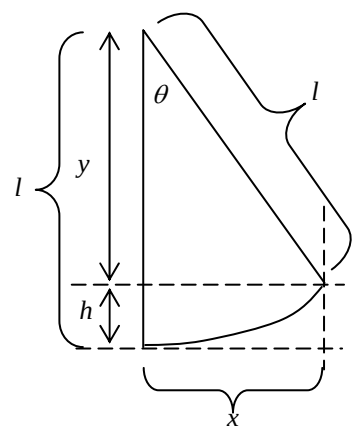

Fig 8.3

But, the point is to find the velocity of the bullet, so this problem is actually done backwards. The length of the rope holding the pendulum is known (l) and the distance the pendulum moves in the x direction (x) is measured (or the angle of swing, θ, is measured). You must use the geometry of a pendulum swing diagramed in Fig 8.2 to find the height (h) that the pendulum rises to $\boxed{y = \sqrt{l^2 - x^2}}$ and $\boxed{l = y + h}$, so $\boxed{h = l - y}$. Plug this final h into $\boxed{\frac{1}{2}mv_0^{\ 2} = mgh}$ and solve for the initial v_0 of the energy phase. Then recognize that this is the same as v_f for the collision $\boxed{m_b v_{bi} + m_B v_{Bi} = (m_b + m_B)v_f}$ Solve for v_b.

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