1–3 Motion in Two Dimensions

Relative Velocity: Motion in two dimensions, both at constant velocity. A common example is that of a boat crossing a river. In figure 3.1, a boat leaves perpendicular to the shore with a velocity of v_B. The rivers current, v_R, carries it downstream a distance y. In figure 3.2 the boat aims at an angle of θ upstream in order to end up straight across. There is a triangle formed by the solid velocity vectors, and another formed by the dashed displacement vectors. These triangles are similar triangles. The resultant velocity of the boat will be a combination of the boats own velocity and current. This vector is labeled Σv. This is how fast the boat will appear to be moving as seen from a stationary observation point on shore. When calculating the time to cross the stream use the velocity vector and displacement vector that point in the same direction. In figure 3.1, $v_B = \dfrac{x}{t}$. In figure 3.2, $\sum v = \dfrac{x}{t}$

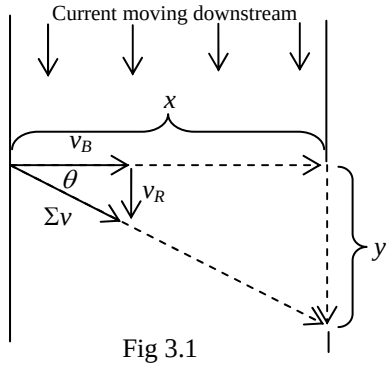


Fig 3.1

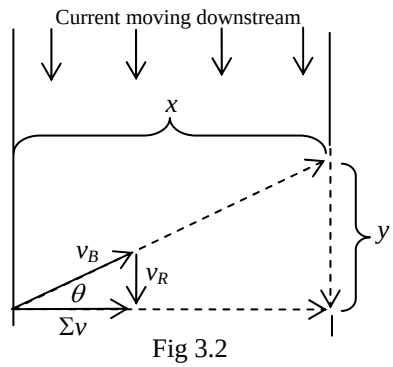


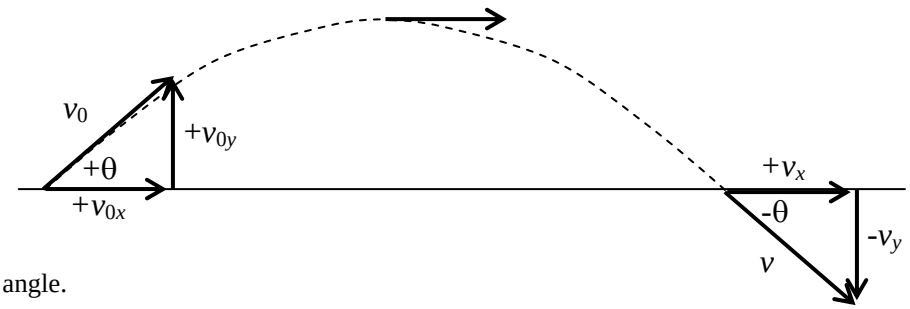
Fig 3.2

Projectile Motion: Motion in one dimension involves acceleration, while the other is at constant velocity.

- In the x-direction the velocity is constant, with no acceleration occurring in this dimension.
- In the y-direction the acceleration of gravity slows upward motion and enhances downward motion.

Vector Components in Projectile Motion: **The x-direction and the y-direction are independent of each other.**

Fig 3.3



Now v_0 is at an angle.

Solve for v_{0x}: $\boxed{v_{0x} = v_0 \cos\theta}$ then use v_{0x} in the kinematic equations to solve for v_x.

Solve for v_{0y}: $\boxed{v_{0y} = v_0 \sin\theta}$ then use v_{0y} in the falling body equations to solve for v_y.

v_x and v_y are component vectors. To find v, use Pythagorean Theorem $\boxed{v = \sqrt{v_x^{\,2} + v_y^{\,2}}}$ and arctangent $\boxed{\theta = \tan^{-1}\dfrac{v_y}{v_x}}$

The highest point in the flight: $\boxed{v_x = v_{0x}}$ and $\boxed{v_y = 0}$. If the problem ended here these conditions would apply.

Horizontal Launches

The launch angle is $\boxed{\theta = 0^o}$

$$v_{0x} = v_0 \cos\theta = v_0 \cos 0^o = v_0 \qquad \boxed{v_{0x} = v_0}$$

$$v_{0y} = v_0 \sin\theta = v_0 \sin 0^o = 0 \qquad \boxed{v_{0y} = 0}$$

The above math is not really necessary. Inspection of the Fig 3.4 shows that v_0 is directed straight down the x-axis with no y-component vector visible at all. The end of the problem is similar to the problem depicted in Fig 3.3, above.

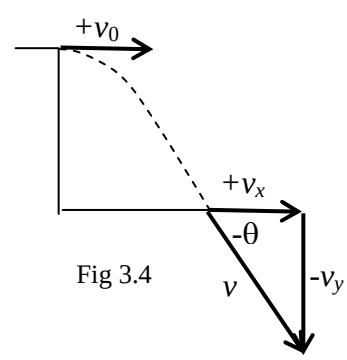


Fig 3.4

Coordinate Axis System provides the necessary orientation to handle the following variables and their appropriate signs: launch angle, initial velocities in x & y, final velocities in x & y, final landing height, and final overall velocity. Orientation matters and thus the coordinate axis becomes a powerful tool, as depicted on the next page.

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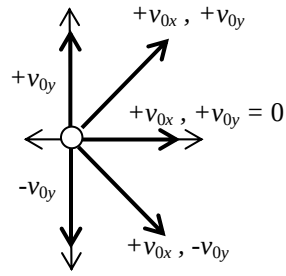
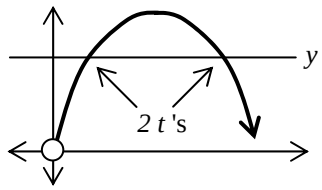
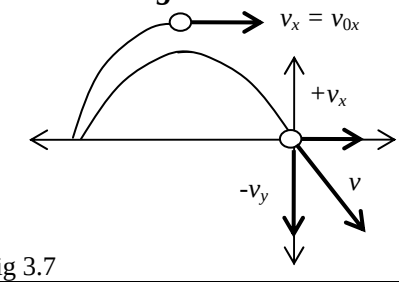
Initial Launch	***During the problem***	***Final Landing***
 Fig 3.5	(At the top $v_x = v_{ox}$ and $v_y = 0$)  Fig 3.6	 Fig 3.7
Initial displacement $x_0 = 0$ $y_0 = 0$ Falling bodies: $\theta = \pm 90^o$ $v_{0x} = v_0 \cos 90^o = 0$ $v_{0y} = v_0 \sin 90^o = v_0$ Horizontal launch: $\theta = 0^o$ $v_{0x} = v_0 \cos 0^o = v_0$ $v_{0y} = v_0 \sin 0^o = 0$ 1st quadrant launch: $+\theta$ $v_{0x} = v_0 \cos\theta$ will be + $v_{0y} = v_0 \sin\theta$ will be + 4th quadrant launch $-\theta$ $v_{0x} = v_0 \cos\theta$ will be + $v_{0y} = v_0 \sin\theta$ will be –	If it lands at the same height as it started ($y = y_0$), then $t_{up} = t_{down}$. There are two t's for every y. The shorter t is for the upward trip. The longer t is for the downward trip. Solve for maximum height two ways 1. From ground up where $v_y = 0$. $v_y^2 = v_{0y}^2 + 2g(y - y_0)$ 2. Or the easy way. Start at the top and pretend it is a falling body. v_x doesn't matter since time is controlled by the y-direction. And at the top v_{oy} is zero. However, this solves for half of the total flight. $y = \frac{1}{2}gt^2$ ***<u>Must double time!</u>***	$+x$ Always $-y$ Lands lower than y_0 $y = 0$ Lands same height as y_0 $+y$ Lands higher than y_0 $a_x = 0$ No a in the x-direction $v_x = v_{0x}$ What is it doing at the end of the problem in the y-direction? $\boxed{\pm v_y}$ It is usually moving downward at the end of the problem. So v_y is ***<u>usually negative</u>*** The final $\boldsymbol{v}$ must be resolved. $v = \sqrt{v_x^2 + v_y^2}$ If $y = y_0$ $v_y = -v_{0y}$ & $v = v_0$

Projectile Motion Strategies

1. ***Horizontal Launch:*** Since $v_{0y} = 0$ and $v_{0x} = v_0$, then use $\boxed{y = 1/2\ gt^2}$ and $\boxed{x = v_{0x}t}$.
2. ***When time (t) or range (x) is given:*** Start with $\boxed{x = v_{0x}t}$ and then $\boxed{y = y_0 + v_{0y}t + 1/2\ gt^2}$.
3. ***No x and no t:*** Time is the key to falling body & projectile problems. Two strategies are useful when time is missing.

1st $y = y_0 + v_{0y}t + 1/2\ gt^2$ 2nd Quadratic Equation 3rd $\boxed{x = v_{0x}t}$	1st $\boxed{v_y^2 = v_{0y}^2 + 2g(y - y_0)}$ solve for v_y which is usually ***<u>negative</u>***. 2nd $\boxed{v_y = v_{0y} + gt}$ use $-v_y$ from above to get t. 3rd $\boxed{x = v_{0x}t}$ use t from above to solve for range x.

Other Projectile Motion Facts

Any ***two launch angles that add to 90°*** will arrive at the same landing site if fired on level ground. Examples: 15° and 75°, 30° and 60°, and 40° and 50°. ***Maximum range*** (maximum distance in the x-direction) is achieved by launching at 45° (45° + 45° = 90°). ***Maximum altitude*** (maximum distance in the y-direction) is achieved by firing straight up, at 90°.

Circular Motion

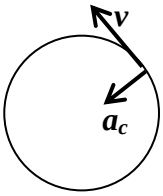


Fig 3.8

Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution, $\boxed{T = \frac{1}{f}}$. Time is in the numerator.

Velocity: In uniform circular motion the magnitude (speed) of the object is not changing. However, the direction is constantly changing, and this means a change in velocity (a vector composed of both magnitude and direction). In circular motion one can describe the rate of motion as either a speed or as a **tangential velocity** $\boxed{v = \frac{2\pi r}{T}}$. This velocity is an instantaneous velocity and it is directed tangent to the curve.

Centripetal Acceleration: The object is continually turning toward the center of the circle, but never gets there due to its tangential velocity. This centripetal (center seeking) change in velocity, is a centripetal acceleration $\boxed{a_c = \frac{v^2}{r}}$

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