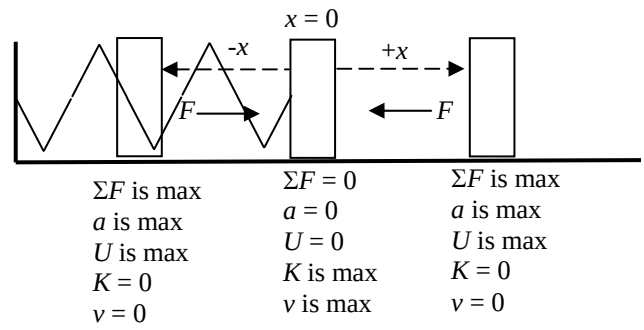
1–7 Oscillations

Period: $\boxed{T = \frac{1}{f}}$ Time for one revolution.

Frequency: The number of revolution, vibrations, oscillations, or rotations per second.

Springs: A Simple Harmonic Oscillator (SHO)



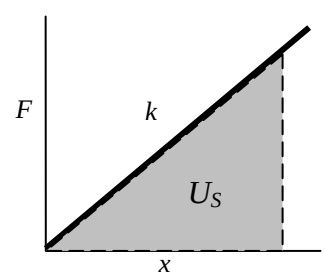
Restoring force: A spring attached to a block is shown at the right. Equilibrium ($x = 0$) is the rest position when no external forces are applied. If the spring is compressed so the block moves to $-x$ or the spring is stretched so the block moves to $+x$ the spring will have a force directed toward the equilibrium position. This is the restoring force $\boxed{F = -kx}$, and is known as Hooke's Law. ***k*** is the spring constant reflecting the quality or strength of the spring. The minus sign shows that the restoring force is opposite the displacement. Move a spring right and it wants to restore to the left. Restoring force is highest at maximum displacement (amplitude), located at $+x$ and $-x$. At these positions the spring has the highest force and acceleration, but has an instantaneous velocity of zero. It also has the highest amount of spring potential energy.

Potential Energy: The energy of position. This time we are working with a spring's position, $\boxed{U_s = \frac{1}{2}kx^2}$. When a spring is at equilibrium it is at rest and has zero displacement. This position is thus has zero potential energy. This is just like gravitational potential energy. When an object is a rest on the ground it is said to have zero potential energy. When it is displaced against the force of gravity it gains potential energy. It wants to return to the ground. The mass is displaced against the force of the spring, which wants to return it to equilibrium. So a displaced spring has stored energy, also known as potential energy. When the spring is fully displaced at either $-x$ or $+x$ it comes to rest.

Amplitude: The distance from equilibrium to the point of maximum displacement. This is the distance form $x = 0$ to $-x$, or from $x = 0$ to $+x$. It is not the distance from $-x$ to $+x$. That distance is two amplitudes. ***How many amplitudes does a spring go through in one complete cycle, from -x to x = 0 to +x, and then back to x = 0 and finally back to -x? Four.***

When the spring reaches $-x$ or $+x$ there it comes to an instantaneous stop, so it has no kinetic energy, but it has potential energy and a very large restoring force. When it moves through the equilibrium position the force on the spring and acceleration are both zero, but the velocity and kinetic energy is now at a maximum. Here any potential energy stored by displacing the spring from rest is turned into kinetic energy. Conservation of energy dictates that if one form of energy disappears and another appears, then the two energies must be equal. So $U_S = K$, or $\boxed{\frac{1}{2}kx^2{}_{at\ either\ end} = \frac{1}{2}mv^2{}_{at\ the\ middle}}$

Springs are a variable force. $F_s = -kx$ The more a spring is displaced, x, the larger the force, F, that is needed to displace it. At first no force is needed to move the spring. At the end of the problem a force F_S is needed. So the average force needed during the entire displacement is $\bar{F}_s = \frac{F_s - 0}{2} = \frac{1}{2}F_s = \frac{1}{2}kx$. Work is



$W = F \cdot d = \frac{1}{2}kx \cdot x = \boxed{\frac{1}{2}kx^2}$. Force and displacement can be graphed as shown at the right. The equation $F_s = kx$ is the equation for a line $y = mx + b$, where k is the slope.

U_S is the area under the curve, $\boxed{\frac{1}{2}bh} = \frac{1}{2}(x)(F_s) = \frac{1}{2}(x)(kx) = \boxed{\frac{1}{2}kx^2 = U_s}$

Period of a spring: Time required for one oscillation, $\boxed{T_s = 2\pi\sqrt{\frac{m}{k}}}$. Depends on mass of object attached to spring and k.

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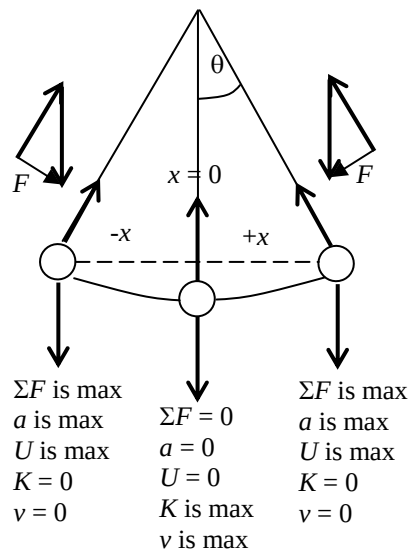
Pendulums: Approximates a Simple Harmonic Oscillator (SHO)

Restoring force: The restoring force is the sum of the tension and gravity vectors diagramed to the right. When the object is at its extreme displacement ($+x$ or $-x$) the restoring force is greatest. When the object returns to the equilibrium position ($x = 0$) the sum of the vectors is zero, and there is no restoring force. This is just like the spring above. There is never any net force at the equilibrium position ($x = 0$), so there is no acceleration at this point. Both the pendulum and the spring move through the equilibrium by inertia. It is interesting to note that when the restoring force is highest (at maximum displacement, either $+x$ or $-x$), the velocity is zero. When the net force is zero (equilibrium position, $x = 0$), the velocity is at its highest. All of the proceeding is true for any harmonic oscillators. Note: the dashed displacement line is not the same length as the arc that a pendulum actually moves through. The difference is very small at angles less than or equal to 15°. Beyond this angle pendulums are not good harmonic oscillators. So at angles under 15° pendulums behave almost like a true SHO.



Energy: The pendulum has potential energy at $+x$ and $-x$. It has kinetic energy at $x = 0$. The potential energy at one either end equals the kinetic energy in the middle. Potential energy in a pendulum has to do with height, not the spring constant $\boxed{mgh_{at\ either\ end} = \frac{1}{2}mv^2{}_{at\ the\ middle}}$

Period: Time required for one oscillation, $\boxed{T_p = 2\pi\sqrt{\frac{\ell}{g}}}$. Depends on length of the pendulum and $\boldsymbol{g}$.

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