1–5 Applications of Force

Strategy for Force Problems

1. Draw ***FBD***.
2. Decide direction of motion. Considered this the positive direction. ***If the object does not move, ask which direction it would move if it were free to do so, and set this as the positive direction.***
3. **Which direction matters, the *x* or the *y*-direction? What is it doing in the direction that matter?**
4. Construct $\sum F$ equations in the relevant direction, by looking at the ***FBD***. Any force vectors or components of force vectors pointing in the direction of motion are positive while any vectors or components opposing motion are negative.
5. Substitute known equation, $\sum F = ma$, $F_g = mg$, from the table on page 6, into the sum of force equation.
6. Plug in values and solve. All values including 9.8 are positive since the plusses and minuses have already been decided.

Example 5-1: Static Force Problems, and Force Triangles

In beginning physics many force problems contain a force triangle.

A mass is suspended by two strings from the ceiling, as shown in Fig 5.1a. Fig. 5.1b is the FBD. In Fig 5.1c the tensions are separated into *x and y* components. Sum the forces in the *x* and *y*-directions separately. The sum of force will be equal to zero.

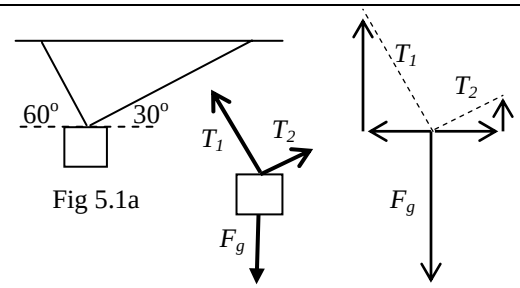

Fig 5.1a Fig 5.1b Fig 5.1c

$$\sum F_x = T_{2x} - T_{1x} \qquad \boxed{T_{1x} = T_{2x}}$$

$$\sum F_y = T_{1x} + T_{2y} - F_g \qquad \boxed{T_{1x} + T_{2y} = F_g}$$

Under the right circumstances the three force vectors form a right triangle when added tip to tail. Note the 30° and 60° angles above. The forces complete the 30-60-90 triangle shown in Fig 5.1d. The sum of force is zero. Tensions can be calculated using SOH CAH TOA and Pythagorean Theorem.

$$\boxed{T_1 = F_g \cos 30^o} \; \boxed{T_2 = F_g \sin 30^o} \quad \text{or} \quad \boxed{T_1 = F_g \sin 60^o} \; \boxed{T_2 = F_g \cos 60^o}$$

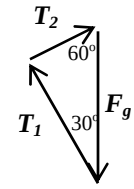

Fig 5.1d

Example 5-2: Lawn Mower

An old fashioned lawnmower is pushed with 90N at a 45° angle against a horizontal retarding force. Fig 5.2a is the FBD, while Figure 5.2b is a diagram of horizontal and vertical component vectors

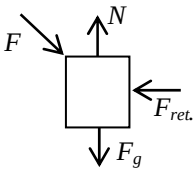

Fig 5.2a

Solve for the Retarding Force

$$\sum F_x = F_x - F_{ret.} \qquad 0 = F_x - F_{ret.} \qquad F_{ret.} = F_x = 90N\cos 45^o = \boxed{63.6N}$$

Solve for the Normal Force

$$\sum F_y = -F_y + N - F_g$$

$$N = \sum F_y + F_y + F_g \qquad N = 0 + \left(90N \sin 45^o\right) + \left[(16kg)\left(9.8\,m/s^2\right)\right] = \boxed{220N}$$

Solve for F to accelerate from rest to 1.5 m/s in 2.5 s

$$v_x = v_{x_o} + a_x t \qquad a_x = \frac{v_x - v_{x_o}}{t} = \frac{1.5 - 0}{2.5} = 0.6\,m/s^2 \qquad \sum F_x = ma_x = (16kg)\left(0.6\,m/s^2\right) = \boxed{9.6N}$$

You need this force to accelerate, but you still need to overcome the retarding force.

$$\sum F_x = F_x - F_{ret.} \qquad F_x = \sum F_x + F_{ret.} = 9.6N + 63.6N = 73.2N$$

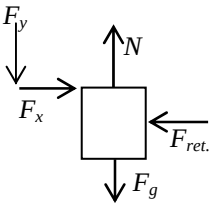

Fig 5.2b

But you aren't pushing in the *x*-direction. You need the push at 45° to generate 73.2 *N* in the *x*-direction.

$$F_x = F\cos 45^o \qquad F = \frac{F_x}{\cos 45^o} = \frac{73.2N}{0.707} = \boxed{104N}$$

Apparent Weight: When you ride in an elevator upward you will feel heavier when the elevator accelerates and lighter as it slows to a stop. When riding downward you will feel lighter when it accelerates and heavier when it stops. When you ride a roller coaster you experience the same sensations when moving up and down. On "Superman the Ride" you can actually feel weightless. Astronauts and pilots experience these same sensations when moving away or toward the center of the earth (or the moon). Astronauts also feel weightlessness as well. This effect is not limited to the vertical or *y* direction.

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When you "step on it" in a car you feel yourself pressed into the seat, and when you panic stop you feel yourself thrown forward. These sensations have the same characteristics as gravity or weight. Up to now we have analyzed the motion of the car, the plane, the rocket, etc. using straight forward force and kinematics. <u>*With apparent weight we are dealing with a false force that the passenger feels*</u>. The effect is really created by the passenger's inertia. When you "step on it" you don't sink into the car seat. You actually follow inertia and stay at rest, while the car hits you from behind. The real acting forces are the opposite of what our brain thinks. ***To analyze this feeling of force for a passenger we need apparent weight***. Apparent weight is the weight that would show on a bathroom scale if you were between you and the surface causing the force. Contact with any surfaces is a normal force and bathroom scales measure normal force. So weight apparent is also F_N.

$\boxed{F_{g\,apparent} = mg \pm ma}$ This equation adds the acceleration of a passenger's vehicle to the real weight.

x* direction:** $F_{g\,apparent\ x} = mg_x \pm ma_x$, but with no ***g in the ***x*** direction $\boxed{F_{g\,apparent\ x} = \pm ma_x}$

Positive: accelerating, you feel heavier. Negative: decelerating, you feel lighter.

***y* direction:** $\boxed{F_{g\,apparent\ y} = mg_y \pm ma_y}$, so acceleration adds / subtracts from the real / actual weight.

Positive: moving away from the center of gravity (up or away from Earth). Negative: moving toward the center of gravity.

Why do you feel weightless on "Superman the Ride"? Because, the acceleration is downward and matches gravity.

$$F_{g\,apparent\ y} = mg_y \pm ma_y = m(9.8) - m(9.8) = 0\,m/s^2$$

g's: The acceleration of gravity can also be expressed in g's. $\boxed{1g = 9.8\,m/s^2}$ This is commonly used in flight terminology.

Example 5-3: Compound Bodies, One Dimension

These problems have two or more masses connected by a string or pressed against each other.

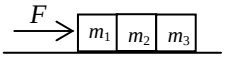


Fig 5.3a

In Fig 5.3a a force F presses against block 1 which presses with a normal force against block 2 which then presses with a normal force on block 3.

If all the blocks are the same mass what is the acceleration of the blocks? Simply treat all the masses as one larger block, and remember to use the sum of force to find acceleration.

$$\sum F_{system} = F \qquad m_{system}a = F \qquad (m_1 + m_2 + m_3)a = F \qquad \boxed{a = \frac{F}{(m_1 + m_2 + m_3)}}$$

How much force F is acting on each block? The force will distribute proportionally based on mass. If the blocks are of equal mass then the force on each of the three blocks will be one third. $\boxed{F_1 = \frac{F}{3}}$ $\boxed{F_2 = \frac{F}{3}}$ $\boxed{F_3 = \frac{F}{3}}$

If the blocks do not have the same mass then you must distribute the force using the mass ratio.

How much force is acting at the boundary between the blocks? This force is a normal force as it is created by surface contact. Block 1 requires one third of F to move. So two thirds of F remain to push blocks 3 and 4. The force at the boundary between blocks 1 and 2 is two thirds F, and this is the force needed to push the blocks behind the boundary. The last block C only requires one third of force F and thus the force at the boundary between blocks B and C is one third F. This may make more sense when examined vertically. ***If the same blocks are stacked on a table one could imagine that they are gymnasts standing on each other's shoulders.*** How much of the total force F is felt on the bottom gymnasts feet? All of the force. How much is at the boundary? The boundary can be analyzed from either surface of contact. If we look at the bottom gymnasts shoulders, he must push up with 2/3 F to hold 2/3 m. The middle gymnasts feet must support his own weight and that of the gymnast above, so his feet must push with 2/3 F. No matter how you look at the boundary you arrive at the same answer. The boundary between the middle and top gymnast is 1/3 F, as the middle gymnast shoulder and top gymnasts feet only need to support the top gymnasts 1/3 m.

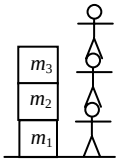


Fig 5.3b

The problem is exactly the same if the blocks are tied by strings and hanging from the ceiling. Only now we use tension instead of force normal, and the gymnasts are hanging from a cliff. If the top rope (top arm) in Fig 5.3c has a tension of F, what is the tension between blocks 1 and 2? 2/3 F. The top gymnast lower arm and the middle gymnasts upper arm both have to support 2/3 m. How much tension is in the rope between blocks 2 and 3? 1/3 F

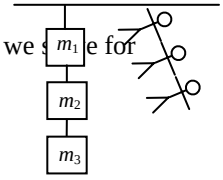
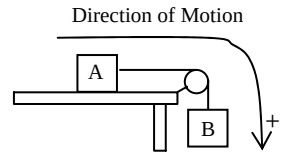
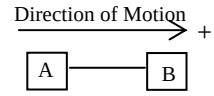


Fig 5.3c

Compound Bodies in Two Dimensions: These problems have two or more masses connected by a strings and involve pulleys. Pulleys are devices that change the direction of force. The pulleys in the following examples will be considered massless and frictionless, and as a result they do not change the magnitude of force. As two dimensions are involved simultaneously the assignment of positive and negatives on the various forces can be tricky. The easiest method to achieve sign consistency throughout a complex problem is to identify the direction the masses are moving, or the direction they are most likely to move. ***Set the direction of motion as positive***, as shown in the top figure to the right. Every vector in the direction of motion will be considered positive. Those opposed to the direction of motion are negative. If you are not sure what the direction of motion will be, take a guess. If you calculate a negative value for acceleration, you were wrong, the masses actually moved in the direction opposite your prediction. ***A very useful shortcut is to reorient the problem into one dimension.*** Pulleys are devices that change the direction of force. So pretend the pulley is not there, as shown in the figure at the bottom right.





Example 5-4: Compound Body Moving in Two Dimensions

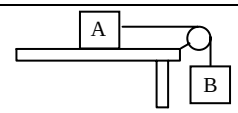


Fig 5.4a

Solve for acceleration: Fig 5.4a shows the scenario. As there are two masses there are two FBD's shown in Fig 5.4b. Fig 5.4c is an informal sketch of connected boxes. Use this sketch and the combined mass method to solve for overall acceleration. ***Remember, when you use the combined method you must total all the masses for the sum of force***. F_g and F_N acting on mass A are perpendicular to motion. They cancel each other. Fig 5.4c shows that the ***tension in the rope also cancels***. It is the same rope so the value at both ends is the same and the direction of tension is opposite. So tension cancels in the shortcut to find acceleration.

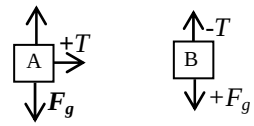


Fig 5.4b

$$\sum F_{AB} = F_{gB} \qquad (m_A + m_B)a = m_B g \qquad \boxed{a = \frac{m_B g}{(m_A + m_B)}}$$

Solve for the tension in the rope: In order to solve for tension you need a formula with tension in it. You must solve for one of the masses by itself. Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

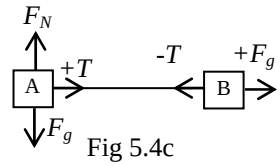


Fig 5.4c

$$\sum F_A = T - F_{frA} \quad \text{or} \quad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + \mu m_A g} \qquad \boxed{T = m_B g - m_B a}$$

Plug the acceleration from part one into either equation above and you will get the same final answer.

Example 5-5: Atwood Machine:

Atwood created a device to artificially slow the acceleration of gravity. In Fig 5.4a it doesn't say which mass is greater. I picked the two masses B and C as the more massive side and used this to set the direction of motion.

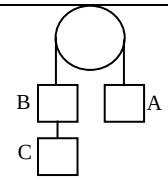


Fig 5.5a

Solve for acceleration: The FBD's for all blocks are shown in Fig 5.5b. Use the combined mass method to solve for overall acceleration. *Remember, when you use the combined method you must total all the masses for the sum of force*. In addition it is easier if you treat blocks B and C as though they are one larger block having a single mass. Fig 5.5c is a sketch of the masses as a linear problem, with the left masses combined.

$$\sum F_{ABC} = F_{gBC} - F_{gA}$$

$$(m_A + m_B + m_C)a = (m_B + m_C)g - m_A g \qquad a = \frac{(m_B + m_C)g - m_A g}{(m_A + m_B + m_C)}$$

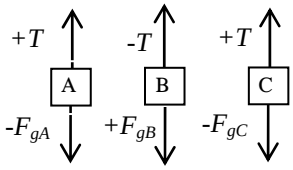


Fig 5.5b

If asked for the tension in the rope connecting mass A and B you must sum the forces for any block connected to the rope. A problem might only give information for one of the two blocks, or one of the blocks will be much simpler to solve. Learn to identify the easy block. If more than one mass is suspended by a rope, then add the masses suspended by the rope. This is the case for blocks B and C. Both of the possible solutions are detailed, one using block A on the left, and the other using blocks B and C to the right.

$$\sum F_A = T - F_{gA} \quad \text{or} \quad \sum F_{BC} = F_{gBC} - T$$

$$T = \sum F_A + F_{gA} \qquad T = F_{gBC} - \sum F_{BC}$$

$$\boxed{T = m_A a + m_A g} \qquad \boxed{T = (m_B + m_C)g - (m_B + m_C)a}$$

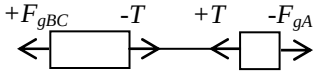


Fig 5.5c

Plug in the acceleration from above to solve for F_T. To solve for the tension in the rope between block B and C, just use the mass of block C. Refer to the right most FBD in Fig 5.5b.

$$\sum F_C = F_{gC} - T \qquad T = F_{gC} - \sum F_C \qquad \boxed{T = m_C g - m_C a}$$

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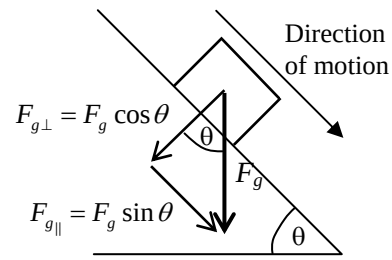
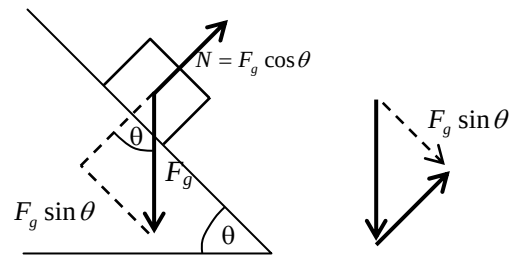
Force Gravity on Slopes: Motion on a slope is parallel to the slope. F_g is then at an angle to this motion. Any vector at an angle to motion should be split into components parallel and perpendicular to the chosen orientation (direction of motion). In this case F_g should split into $F_{g\parallel}$ and $F_{g\perp}$ to the slope. F_g is actually pulling the block into the slope $F_{g\perp} = F_g \cos\theta$ and down the slope $F_{g_\parallel} = F_g \sin\theta$.



But the block is not moving perpendicular to the slope, so the sum of forces in the perpendicular direction is zero. If gravity is pulling the block into the slope then the slope must push back with an equal and opposite force. Any force involving contact with a surface is call a **Normal Force, *N***. In slope problems $\boxed{N = F_g \cos\theta}$.

When a block moves on a flat surface the slope angle is 0°. Plug 0° into the N equation and it reduces to $\boxed{N = F_g}$. This is an abbreviation for N that works only on flat surfaces. The component of gravity parallel to the slope $\boxed{F_{g\parallel} = F_g \sin\theta}$ is then left over. $F_{g\parallel}$ is the resultant when F_g and N are added by vector addition. This is the force of gravity that can accelerate objects down slopes.



Example 5-6: Combined Bodies and Slopes

Solve for acceleration: This is just like Example 5-4, only this time block A is on a slope. Just use the same problem solving approach and recognize that gravity is different on slopes. Fig 5.7a diagrams the problem. Fig 5.7b shows the FBD's for the two masses. Make a note of the orientation of the vectors for block A. Fig 5.6c depicts the problem as a linear problem. As usual gravity is pulling block B, and as it is felt entirely in the y direction, it is acting with full strength. However, gravity is acting at an angle to the motion of block A on the slope. ***We need the component of gravity acting in the direction of motion, $F_g \sin\theta$, in order to solve the problem.***

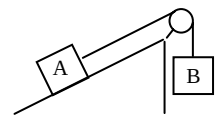


Fig 5.6a

$$\sum F_{AB} = F_{gB} - F_{gA} \sin\theta$$

$$(m_A + m_B)a = m_B g - m_A g \sin\theta \qquad \boxed{a = \frac{m_B g - m_A g \sin\theta}{(m_A + m_B)}}$$

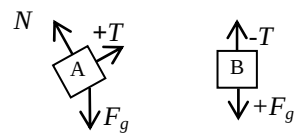


Fig 5.6b

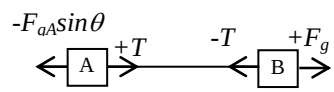


Fig 5.6c

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\sum F_A = T - F_{gA} \sin\theta \quad \text{or} \quad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + m_A g \sin\theta} \qquad \boxed{T = m_B g - m_B a}$$

Plug the acceleration from part one into either equation above and you will get the same final answer.

Friction: Opposes motion and is always negative. Motion is always parallel to a surface, so friction always acts parallel.

Static Friction: Friction that will prevent an object from moving. As long as the object is standing still the force of friction must be equal to the push, pull, component of gravity or other force that attempts to move the object. (If there is no force attempting to cause motion, then there can be no friction). ***Oddly enough the maximum value for static friction is measured just as the object breaks loose and begins to move.*** *Static friction is the strongest friction since the surfaces have a stronger adherence when stationary.*

Kinetic Friction: Friction for moving objects. Once an object begins to move breaking static frictions hold, then the friction is termed kinetic. Kinetic friction is not as strong as static friction, but it still opposes motion.

Coefficient of friction: μ is a value of the adherence or strength of friction. μ_k for kinetic and μ_s for static.

$\boxed{f = \mu N}$ so $\boxed{f = \mu mg \cos\theta}$ On flat surfaces $\theta = 0^o$, $\boxed{f = \mu mg}$

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Example 5-7: Two Dimension Compound Body with Friction

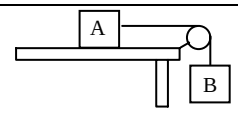


Fig 5.7a

Solve for acceleration: This is a repeat of Example 5-4, only this time friction appears in the FBD for mass A. Friction opposes motion and is therefore negative.

$$\sum F_{AB} = F_{gB} - F_{frA} \qquad (m_A + m_B)a = m_B g - \mu m_A g \qquad \boxed{a = \frac{m_B g - \mu m_A g}{(m_A + m_B)}}$$

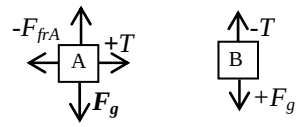


Fig 5.7b

Solve for the tension in the rope: Solve for either body. On tests choose the easy one, this will usually be the hanging mass.

$$\sum F_A = T - F_{frA} \qquad \text{or} \qquad \sum F_B = F_{gB} - T$$

$$\boxed{T = m_A a + \mu m_A g} \qquad \boxed{T = m_B g - m_B a}$$

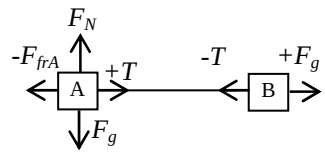


Fig 5.7c

Plug the acceleration from part one into either equation above and you will get the same final answer.

Example 5-8: A Complex Slope Problem

Objects can move up or down a slope. $\boxed{F_g \sin\theta}$ is simply the component of gravity pulling on an object down the slope in a direction parallel to the slope. In Fig 5.8a a man is pushing mass B up a slope. In this case gravity is opposite the direction of motion, as is friction. The FBD's for both blocks are shown in Fig 5.8b, with the main difference being the presence of the true forces, F_g and F_N instead of $F_g \sin\theta$. Remember, F_g and F_N are summed to create $F_g \sin\theta$. The direction of motion (positive direction) is up the slope.

$$\sum F_A = F_{gA} - T \qquad \sum F_B = F + T - F_{gB}\sin\theta - f_B$$

$$T = F_{gA} - \sum F_A \qquad T = \sum F_B - F + F_{gB}\sin\theta + f_B$$

$$F_{gA} - \sum F_A = \sum F_B - F + F_{gB}\sin\theta + f_B$$

$$m_A g - m_A a = m_B a - F + m_B g\sin\theta + \mu m_B g\cos\theta$$

$$m_A a + m_B a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$$

$$(m_A + m_B)a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$$

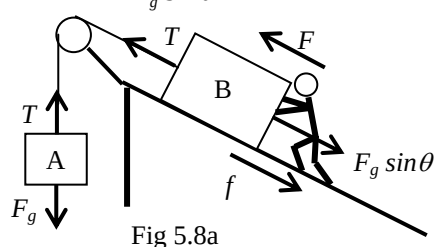


Fig 5.8a

If constant velocity $\boxed{0 = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}$

If accelerating $\boxed{a = \dfrac{m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}{m_A + m_B}}$

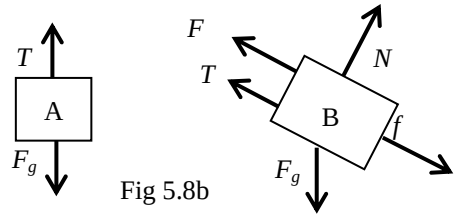
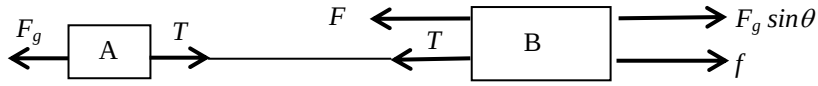


Fig 5.8b

An easier way to solve any compound body problem connected by a string is to stretch it out in a linear manner as shown in Fig 5.8c.

Fig 5.8c



The direction of motion is to the left, so all left arrows are positive, and all right arrows are negative. The string attaching the two boxes is the same and the two equal and opposite tensions cancel each other out. This allows you to solve for an overall Force Net in fewer steps. ***<u>The main difference here is that the mass in the sum of force substitution is the total mass of the whole problem</u>***.

$$\sum F = F_{gA} + F - F_{gB}\sin\theta - f_B$$

$(m_A + m_B)a = m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta$, which is identical to the sixth line in the longer version above.

Constant velocity $\boxed{0 = m_A g + F_P - m_B g\sin\theta - \mu m_B g\cos\theta}$ or accelerating $\boxed{a = \dfrac{m_A g + F - m_B g\sin\theta - \mu m_B g\cos\theta}{m_A + m_B}}$

However, this is not going to solve for the force of tension in the string. You must select one of the masses and work with its FBD. Block A obviously has the simpler of the FBD, so choose it.

$$\sum F_A = F_{gA} - T \qquad T = F_{gA} - \sum F_A \qquad \boxed{T = m_A g - m_A a}$$

If you are lucky the blocks are standing still or moving at constant velocity, in which case a = 0. If not use the a from above.

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