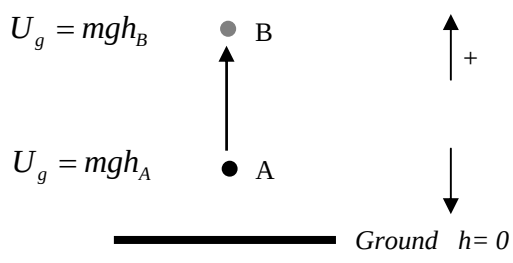
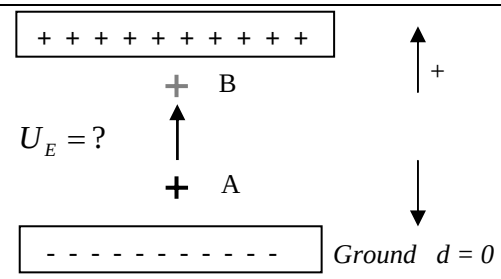
3–16 Electric Potential

Electric Energy Compared to Gravitational Potential Energy: Electric fields created by charged plates (Q) are directed from positive to negative. Electrons ($-q$) or protons ($+q$), which have their own electric fields, experience a force when placed between plates. The positive proton follows the electric field, while the electron moves opposite the field.

Gravitational Fields	Electric Fields
There is only one kind of mass and it only attracts. In the following scenarios a mass will be moved in a gravitational field created by the planet earth.	Two types of charge that repel and attract. ***<u>Electrical theory is based on what positive charges do.</u>*** The following scenarios show a positive charge moved in an electrical field.
	
• Gravitational Field acts from + sky to – ground. • ***<u>Masses naturally move in the field direction.</u>*** • To lift a mass opposite the field direction, from A to B, you must add energy. You must do $+W$. • Work Energy Theorem states that $W = \Delta\, Energy$. • Point A has less potential energy than point B. • Moving from A to B increases U_g. • The increase (change) in U_g equals the work done. ***Masses only attract each other.*** They only attract and thus fall toward each other. But gravity is very weak, billions and billions of times weaker than electricity. <u>So in electric force problems gravity is ignored</u>. It is there, but it is mathematically irrelevant. One exception is the Milikan Oil Drop Experiment, where he placed excess charge on oil drops. The oil drops have significant mass compared to electrons & the gravity on the oil drops (not the electrons) matters.	• Electric Field acts from + plate to – plate. • ***<u>Positive charges naturally move in the field direction.</u>*** • To move positive charge opposite the field direction, from A to B, you must add energy. You must do $+W$. • Work Energy Theorem states that $W = \Delta\, Energy$. • Point A has less potential energy than point B. • Moving from A to B increases U_E. • The increase (change) in U_E equals the work done. ***But, Electricity has positives and negatives.*** Opposite charges attract and positive charge in electric fields act just like masses in a gravity fields. Negative charges follow the same rules but move in the opposite direction. They fall up, so to speak. They are repelled by the negative ground and attract to the positive. In addition to this like charges repel. So while it is similar to gravtiy there is added complexity. The math is the same, but the directions of fields, forces, and velocities can be confusing.
m causes gravity g measure of field strength h distance above ground	q causes electricity E measure of field strength d distance above negative plate (ground)
Gravitational Potential Energy $\boxed{U_g = mgh}$ In Newtonian Mechanics there is no expression similar to Electric Potential. In electricity $V = Ed$, but $? = gh$ The other equation for potential energy of gravity was solved in the mechanics section on gravity, as follows: $W_g = \Delta U_g = F_g \Delta r$. Set the initial displacement as zero and it simplifies to $U_g = F_g r$. Use this with $\boxed{F_g = -G\frac{m_1 m_2}{r^2}}$ to get $\frac{U_g}{r} = -G\frac{m_1 m_2}{r^2}$. This simplifies to $\boxed{U_g = -G\frac{m_1 m_2}{r}}$.	**Electric Potential Energy** $U_E = qEd$ New quantity in electricity $\boxed{E = \frac{V}{d}}$ so $V = Ed$ Substitute $V = Ed$ into the electric potential energy equation and you get its final form $\boxed{U_E = qV}$. Combine $U_E = qEd$ & $E = k\frac{q}{r^2}$ to get $U_E = q\left(k\frac{q}{r^2}\right)d$ r and d cancel since both measure distance. $\boxed{U_E = qV = k\frac{q_1 q_2}{r}}$

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Gravitational Fields	Electric Fields
Work: $W = \Delta U_g = mg\Delta h$	**Work**: $W = \Delta U_E = q\Delta V$
	Electric Potential Due to Several Charges Combine $E = k\frac{q}{r^2}$ & $E = \frac{V}{d}$ to get $\frac{V}{d} = k\frac{q}{r^2}$ r and d cancel since both measure distance. Simplifies to $V = k\frac{q}{r}$ and is written as $\boxed{V = k\sum_i \frac{q_i}{r_i}}$

Electric Potential is easily confused with Electric Potential Energy. Even though they sound similar they are different variables. Unfortunately there are also a number of expressions for each variable. Know the following!
V: ***Potential, Electric Potential, Potential difference, and Voltage***.
U_E: ***Potential Energy, and Electric Potential Energy***.
You can tell if it is $\boldsymbol{U_E}$, as it has the critical word ***Energy***.

Example 16-1: Electric Potential Due to Several Charges

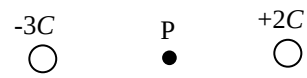


Fig 16.1

What is the electric potential at point P located half way between a –3 *C* charge and a +2 *C* charge separated by 2 *m*, as shown in Fig 16.1?

Potential is not a vector. This time just sum the quantities in a very straight forward manner, using the equation below. ***Since it is not a vector the minus signs on the charge are included in the equation.***

$$V = k\sum_i \frac{q_i}{r_i} \qquad V = 9\times10^9 \frac{N\cdot m^2}{C^2}\left(\frac{-3C}{1m} + \frac{+2C}{1m}\right) = -9\times10^9 \frac{N\cdot m}{C} = \boxed{-9\times10^9 V}$$

Example 16-2: Electric Field, Potential, and Potential Energy

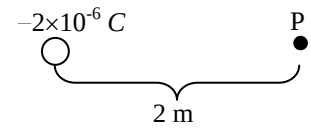


Fig 16.2a

A $+2\times10^{-6}$C charged mass is located 2 m to the left of point P, as shown in Fig. 16.2a

Solve for the Electric Field at point P in Fig 16.2a.

$$E = k\frac{q}{r^2} \qquad E = \left(9\times10^9\right)\frac{\left(2\times10^{-6}\right)}{(2)^2} = \boxed{4.5\times10^3\ N/C,\ \text{left}}$$

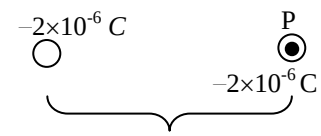


Fig 16.2b

Remember, electric field is a vector. Do not include the minus sign on the, -2C charge, in the math when solving for the magnitude of the field. Use the minus on the charge to establish the direction of the field. Electric field is directed toward negative charges (ground), just as the gravity field is directed downward (-) toward the ground.

Solve for the Electric Potential at point P in Fig 16.2a.

$$V = k\sum \frac{q_i}{r_i} \qquad V = \left(9\times10^9\right)\frac{\left(-2\times10^{-6}\right)}{(2)} = \boxed{-9\times10^3 V}$$

Voltage is not a vector. Just plug in the values, minus signs and all, as you see it. There are no worries about direction.

Another –2 C charge mass is moved to point P, as shown in Fig 16.2b. Solve for the systems Electric Potential Energy.

$$U_E = qV = \left(2\times10^{-6}\right)\left(9\times10^3\right) = \boxed{18\times10^{-2}\ J} \quad \text{or} \quad U_E = k\frac{q_1q_2}{r} = \left(9\times10^9\right)\frac{\left(2\times10^{-6}\right)\left(2\times10^{-6}\right)}{(2)} = \boxed{1.8\times10^{-2}\ J}$$

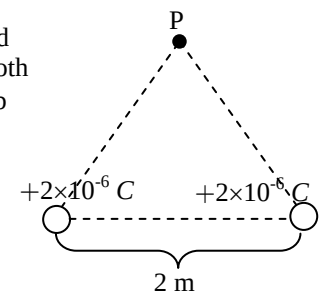
If the charged masses are released from this position, what will their final velocities be? If this potential energy is released the charges will repel and begin moving. The energy of motion is kinetic energy. Try conservation of energy, potential turning into kinetic. ***Just remember there are two charged masses, each having kinetic energy***. This problem will then require mass: both charges are identical with a mass of 3×10^{-12} kg.

$$K_{q1} + K_{q2} = U_E \qquad (2\ \text{charges})\left(\frac{1}{2}mv^2\right) = k\frac{q_1q_2}{r} \qquad v = \sqrt{\frac{kq_1q_2}{mr}} = \sqrt{\frac{\left(9\times10^9\right)\left(2\times10^{-6}\right)\left(2\times10^{-6}\right)}{\left(3\times10^{-12}\right)(2)}} = \boxed{7.75\times10^4\ m/s}$$

This final velocity is reached at infinity, where the potential energy is zero. Can you do this problem in reverse?

Example 16-3: A Two Dimensional *E*, *V*, and U_E Problem

An equilateral triangle is formed by two $+2.0\times10^{-6}$ C charges and point P, as shown in Fig 16.3a. Each side is 2 m long.



Solve for the Electric Field at point. Since this problem involves vectors that are not on the coordinate axis, vector components or a geometric trick must be used. In the component method you will solve for the E_x's and E_y's from each charge, then add them to get the resultants in both the x and y directions. Then Pythagorean Theorem them together to solve for ΣE. Fig. 16.3b shows the vector components needed and this method is shown below

The Long Way

The charges have the same magnitude and are the same distance from P. Therefore, the electric field is the same for both.

$$E = k\frac{q}{r^2} \qquad E_1 = E_2 = \left(9\times10^9\right)\frac{\left(2\times10^{-6}\right)}{(2)^2} = 4.5\times10^3$$

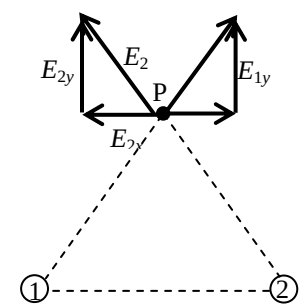


Fig 16.3b

Convert to components.

$$E_{1x} = E_1\cos\theta = 4.5\times10^3\cos60^o = +2250 \qquad E_{1y} = E_1\sin\theta = 4.5\times10^3\sin60^o = +3897$$

$$E_{2x} = E_2\cos\theta = 4.5\times10^3\cos120^o = -2250 \qquad E_{2y} = E_2\sin\theta = 4.5\times10^3\sin120^o = +3897$$

$$\sum E_x = E_{1x} + E_{2x} \qquad \sum E_y = E_{1y} + E_{2y}$$

$$\sum E_x = 2250 - 2250 = 0 \qquad \sum E_y = 3897 + 3897 = +7794$$

Then Pythagorean Theorem and Arctangent the x and y vectors to get the ΣE.

$\sum E = \sqrt{E_x^{\ 2} + E_y^{\ 2}} = \sqrt{(0)^2 + (7794)^2} = 7794\,N/C$ $\qquad \theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{7794}{0}$ this does not solve, but looking at the diagram it is apparent that the two y component vectors are pointing at 90°. $\boxed{7794\,N/C \ \ at\ 90^o}$ or $\boxed{7794\,N/C,\ +y\ direction}$

You wouldn't have known that the direction was $+y$ if you neglected to draw the diagram. In addition, drawing and looking at the diagram reveals an easy shortcut.

Short Way

Both charges are the same, and all distances are the same. So the x-component vectors are equal, as are the y-component vectors. If you imagine all the components added tip to tail it is apparent that the x-components cancel. $\sum E_x = 0$. This means that the final solution is the y-component doubled. $\sum E = E_{1y} + E_{2y} = 2E_y = 2\left(4.5\times10^3\sin60^o\right) = 7794\,N/C$. A glance at the diagram shows that this is pointing in the $+y$-direction. $\boxed{7794\,N/C,\ +y\ direction}$.

<u>Always draw a vector diagram and think about it before you do too much work.</u>

Solve for the Electric Potential at point P in Fig 16.3a.

$$V = k\sum\frac{q_i}{r_i} \qquad V = \left(9\times10^9\right)\frac{\left(+2\times10^{-6}\right)}{(2)} + \left(9\times10^9\right)\frac{\left(+2\times10^{-6}\right)}{(2)} = \boxed{1.8\times10^4 V}$$

Voltage is not a vector. Just plug in the values, minus signs and all, as you see it. There are no worries about direction.

Another +2 C charge mass is moved to point P. Solve for the Electric Potential on this new added charge.

There are three charges so the equation $U_E = k\frac{q_1q_2}{r}$ may not help. Try $U_E = qV$. We just calculated the voltage of pressure, at point P, due to the two original charges. Now a new charge is introduced. The potential energy of this charge is a function of the voltage (from the other charges) pushing on it, and the amount of charge it has.

$$U_E = qV = \left(2\times10^{-6}\right)\left(1.8\times10^4\right) = \boxed{0.036J}$$

This is also the amount of work needed to push this new charge from infinity (zero potential energy) to point P. If this charge were to be released this potential energy would be converted into kinetic energy on this one charge. $U_E = K_{1\,charge}$ **and** $qV = \frac{1}{2}mv^2$**. If all three charges are released then the potential energy converts into kinetic energy of all three, with each receiving one third.** $U_E = K_{3\,charge}$**, so** $U_E = 3K$ **and** $qV = 3\left(\frac{1}{2}mv^2\right)$**.**

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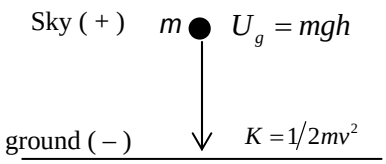
Movement of Mass m/Charge q Parallel and Perpendicular to Gravity / Electric Fields

Movement Parallel to Gravitational Field



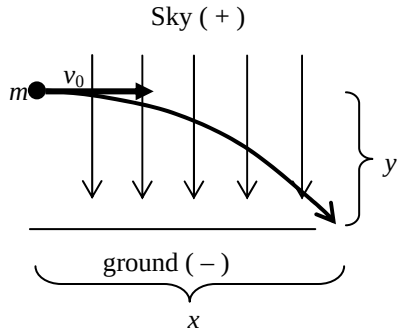
When masses fall from + to –, PE changes to KE

$$mgh = \frac{1}{2}mv^2 \quad (not\ given)$$

Gravity is simple since it is directed toward any mass, and masses are of one type. They fall toward each other.

But, plates are not always up and down like gravity. And there are two types of charges. You must remember that electrical theory is based on positive charge, and that the direction of the electric field is positive to negative (the direction a positive test charge would move). If negative charges are involved then the direction of force and motion are reversed. One of the plates is often depicted with a hole in it. This is the basis of a charge gun. Charges will accelerate toward the plate with the hole, and some will pass through the opening. In our problems the field is assumed to be uniform between the plates (like gravity is assumed to be 9.8 anywhere close to Earth's surface, even though it actually changes slightly with height). And the field is assumed not to extend beyond the plates (It does but is weak). In the example to the right the plates have been rotated 90° reversed and then a negative charge is used. Be prepared for anything!!!!!!!

Projectile Motion



In gravity the acceleration is known, g = 9.8m/s^2

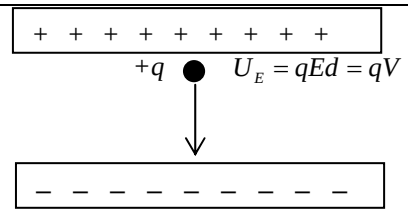
Horizontal launch (as drawn): $v_{0x} = v_0$ and

$v_{0y} = 0$

$$x = x_0 + v_{0x}t + \frac{1}{2}at^2 \qquad y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

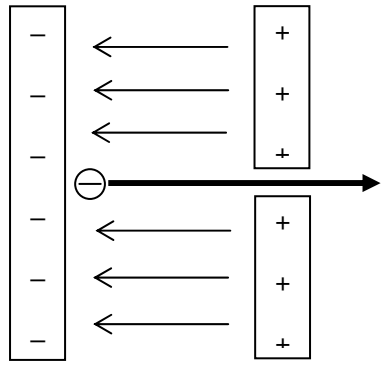
$$x = v_0 t \quad (not\ given) \qquad y = \frac{1}{2}gt^2 \quad (not\ given)$$

Movement Parallel to Electric Field



When positive charges fall from + to –, PE changes to KE

$$qV = \frac{1}{2}mv^2 \quad (not\ given)$$

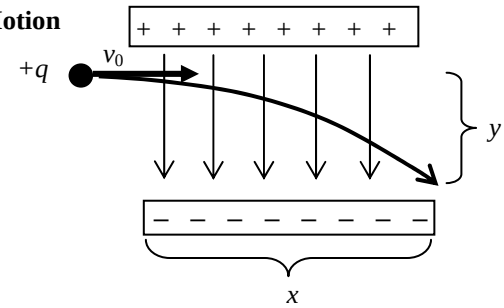


The electric field is directed **– x**, but the negative charge does not follow the field as a positive charge would. It moves instead **+ x**. But the equation is the same.

$$qV = \frac{1}{2}mv^2$$

Note: If the charges are electrons and protons, there will be a difference in final velocities since electrons have less mass than protons.

Projectile Motion



The acceleration of electricity must be solved as it is different for every set of plates.

$$\sum F = F_E \qquad ma = qE \qquad a = \frac{qE}{m}$$

Horizontal launch (as drawn): $v_{0x} = v_0$ and

$v_{0y} = 0$

$$x = x_0 + v_{0x}t + \frac{1}{2}at^2 \qquad y = y_0 + v_{0y}t + \frac{1}{2}at^2$$

$$x = v_0 t \quad (not\ given) \qquad y = \frac{1}{2}at^2 \quad (not\ given)$$

No g (electricity is so strong gravity is ignored). So use we use a which is the acceleration caused by the electric field.

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Example 16.4: Combining the parallel and perpendicular electric fields into one problem

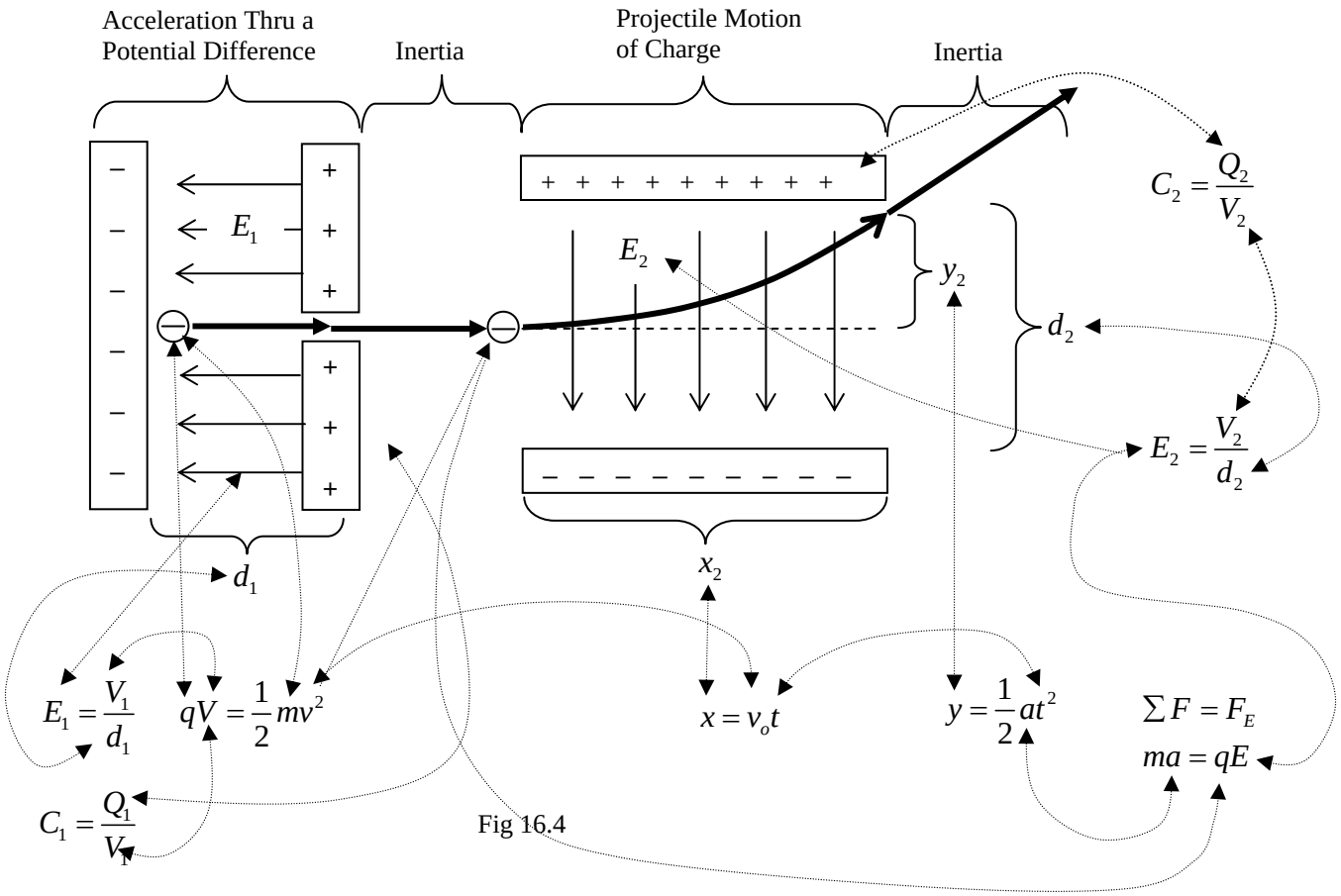

Fig 16.4

In this scenario

An electron is accelerated through a potential difference by the plates to the left. Remember when any field acts parallel to an object it will either accelerate or decelerate the mass or charge. The field is always assumed to be uniform between the plates (same strength everywhere, and it is assumed not to extend beyond the plates). Upon exiting the first set of plates the electron moves at constant velocity due to its inertia. Then it enters a second set of plates. This time the field is perpendicular. Fields that are perpendicular to motion cause either projectile motion or circular motion depending on whether the field is linear or radial. The electron above experience projectile motion due to the linear field, although it appears upside down compared to a gravity problem. The electron continues in a straight path due to its inertia once it leaves the plates.

Given: The electric field E_1 of the first plates and the distance between them d, or the charge on the plates Q_1 and the capictance of the plates C_1.
The mass m_e and charge q_e of an electron accelerated by the plates.
The length x_2 of the second set of plates, the vertical displacement y_2 of the electron, d_2 the distance between the plates.

Find: The electric field E_2 and potential difference V_2 of the second set of plates.
The speed v_f of the electron leaving the second set of plates.

Be able to draw the path of the electron from beginning to end.

Don't Confuse small v for velocity and capital V for voltage.

Try this on for size: What is E_2 in terms of E_1 and any of the other above variables and constants that are relevant?

$$\boxed{E_2 = \frac{4E_1 d_2 y_2}{x_2^{\,2}}}$$

Anything is possible: **The firing:** Plates can be reversed to fire a proton. Plate charge may not be marked, but may be a question. The electric field will not be drawn, instead they will require you to draw the field or to state its direction according to the coordinate axis, Etc. **The deflection:** Plates can be reversed and/or a proton could be used. Plate charge may not be marked, but may be a question. The electric field will not be drawn, instead they will require you to draw the field or to state its direction according to the coordinate axis, Etc.

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