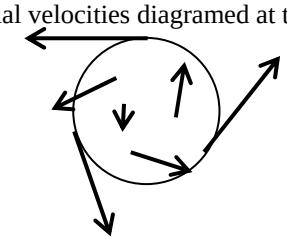
1–10 Introduction to Rotation and Torque

Rotation: In rotation the entire object spins around its center of mass. Looking at the tangential velocities diagramed at the right, we see that they are all in different directions and all vary in magnitude. Points near the outer edge have to move through a larger circumference in the same period than those closer to the center. The outer edge must be moving faster to cover the longer distance in the same period or time. All of these points have one thing in common, they all travel through the same number of degree or radians during a period. Rotational velocity measured in radian per second is called **angular velocity**. However, **All the equations for an object in circular motion hold true if we are looking at a single point and only a specific point on a rotating object.**



Rotating objects have **rotational inertia** and an accompanying **angular momentum**, meaning that a rotating object will continue to rotate (or not rotate) unless acted upon by an **<u>unbalanced</u> torque**, discussed below.

(Note: Planets and satellites follow circular motion, as they are not attached. Inner planets move faster as they are closer to the sun and must have larger tangential velocities. They also travel a shorter circumference. Thus they have shorter periods.)

Angular momentum: Depends on mass (like regular momentum) and it also depends on mass distribution. As an ice skater brings their arms closer to the body they begin to spin faster, since the mass has a shorter distance to travel.

Angular momentum is conserved. The radius gets smaller, but angular velocity increases (vice versa as the skater moves arms outward). A galaxy, solar system, star, or planet forms from a larger cloud of dust. As the cloud is pulled together by gravity its radius shrinks. So the angular velocity must increase. These objects all begin to spin faster. That is why we have day and night.

Torque: In rotation problems we look at the sum of torque (not the sum of force). But it is exactly the same methodology.

$\boxed{\tau = rF\sin\theta}$ Strongest when the force is **<u>perpendicular</u>** to the lever arm (since sin 90° equals one).

Balanced Torque: The sum of torque is zero. No rotation.

Unbalance Torque: Adding all the clockwise and counterclockwise torque does not sum to zero. So there is excess torque in either the clockwise or counterclockwise direction. This will cause the object to rotate.

1. **As always, ask what the object is doing.** Is it rotating or is it standing still?
2. **Set the direction of motion as positive.** The convention when in doubt is that counterclockwise is positive. This corresponds to projectile motion where angles measured from the horizon counterclockwise were positive. But, just like in forces if you know the direction of motion call it positive. It will either rotate clockwise or counterclockwise. If you pick the wrong direction your final answer will be negative. But, the answer will be correct nonetheless. If it is not moving pick one direction to be positive, it really doesn't matter. But the other must be negative, so the opposing torques cancel.
3. **Identify the sum of torque equation.**

$\sum\tau = \sum\tau_{cw} - \sum\tau_{ccw}$ **or** $\sum\tau = \sum\tau_{ccw} - \sum\tau_{cw}$

4. **Substitute the relevant force equations and solve** (examples assume clockwise was positive direction)

Rotating: $\sum\tau = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$

Not Rotating: $0 = \sum(rF\sin\theta)_{cw} - \sum(rF\sin\theta)_{ccw}$ $\quad \sum(rF\sin\theta)_{cw} = \sum(rF\sin\theta)_{ccw}$

Example 10-1: Torque and a Seesaw

Three masses are positioned on a seesaw as shown in Fig 10.1. m_A = 4.0 kg, m_B = 2.0 kg, and m_3 = 3.0 kg. Distances are shown in the diagram.

How far from the fulcrum must m_B be positioned in order for the system to balance? Keep in mind that measurements are made from the center of mass. It is as though all the mass is mathematically located at a point at the center of the object. It is not rotating, so the clockwise torques must equal the counterclockwise torques.

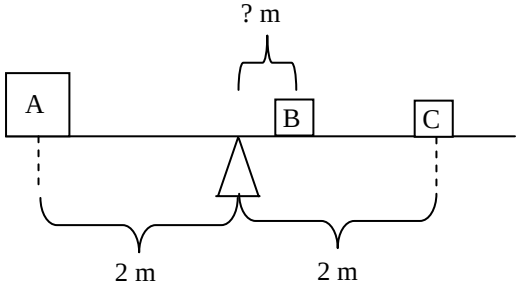


Fig 10.1

$$\sum\tau_{cw} = \sum\tau_{ccw}$$

$$\tau_A = \tau_B + \tau_C$$

$$r_A \cdot F_A = r_B \cdot F_B + r_C \cdot F_C$$

$$r_A \cdot m_A \cancel{g} = r_B \cdot m_B \cancel{g} + r_C \cdot m_C \cancel{g}$$

$$(2)(4) = r_B(2) + (2)(3) \qquad r_B = \boxed{1.0m}$$

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