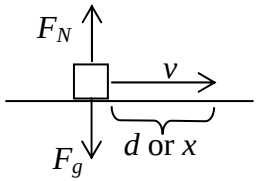
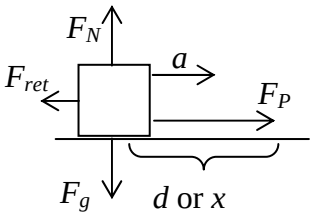
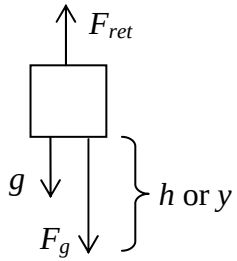
Unit 6 Conclusion

6–29 Kinematics, Force, and Energy Comparison

What is the object doing? What direction is it moving in (if two find *x* and *y* components)? Is it moving at constant *v* (this includes *v* = 0)? Is it accelerating? **What is causing it to do that?** Force? Energy change? See if energy solves the problem first. Then think force and kinematics. Some common motions are discussed in the following pages.

Situation	Kinematics	Force	Energy
Constant Velocity  	Need constant velocity $\boxed{v = \frac{x}{t}}$ or $x = vt$	$\boxed{\sum F = 0}$. Forces are vertical, while motion is horizontal $\boxed{F_g = mg}$ and $\boxed{F_N = mg}$	Inertia only. No force. No energy needed. $W = F \cdot d \cos\theta$ Where θ is the angle between *F* and *d* vectors. $W = F \cdot d \cos 90^o$ $\boxed{W = 0}$
Accelerating in *x*  	$\boxed{v_x = v_{ox} + at}$ $\boxed{v_x^{\ 2} = v_{ox}^{\ 2} + 2a(x - x_o)}$ $\boxed{x = x_o + v_{ox}t + \frac{1}{2}at^2}$ In projectile motion $\boxed{x = v_{ox}t}$	There is a sum of force $\boxed{\sum F = ma}$ $\sum F_x = F_{p_x} - F_{retarding\ x}$ example $\sum F_x = F_{p_x} - F_{fr}$	A force through a distance. $W_x = \sum F_x \cdot d \cos\theta$ Where θ is the angle between *F* and *d* vectors. $W = F \cdot d \cos 0^o$ No retarding forces present $\boxed{W = F \cdot d}$ Now work is done, so there is a Δ energy. $W = \Delta Energy$ $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ The only thing changing is velocity, so *K* is changing.
Accelerating in *y*  	$\boxed{v_y = v_{oy} + gt}$ $\boxed{v_y^{\ 2} = v_{oy}^{\ 2} + 2g(y - y_o)}$ $\boxed{y = y_o + v_{oy}t + \frac{1}{2}gt^2}$ Horizontal projectile or dropped object $\boxed{y = \frac{1}{2}gt^2}$	$\sum F_y = F_g - F_{retarding\ y}$ example $\sum F_y = F_g - F_{air\ resistance}$	$W_y = \sum F_y \cdot h$ Includes retarding forces $W = F_g \cdot h$ No retarding forces $\boxed{W = \Delta U_g = mg\Delta h}$ Height is changing, so *U* is changing. $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ Velocity is changing, so *K* is changing. $mg\Delta h = \frac{1}{2}m(\Delta v)^2$ $\boxed{mgh_i + \frac{1}{2}mv^2{}_i = mgh_f + \frac{1}{2}mv^2{}_f}$

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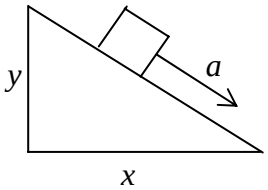
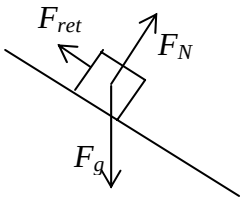
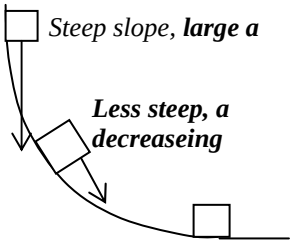
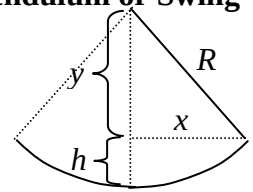
Situation	Kinematics	Force	Energy
Inclines   	Kinematic Equations Apply $v_x = v_{ox} + at$ $v_x^{\,2} = v_{ox}^{\,2} + 2a(x - x_o)$ $x = x_o + v_{ox}t + \frac{1}{2}at^2$ $v_y = v_{oy} + gt$ $v_y^{\,2} = v_{oy}^{\,2} + 2g(y - y_o)$ $y = y_o + v_{oy}t + \frac{1}{2}gt^2$	Motion is parallel to slope Acceleration down the slope is caused by the addition of F_g and F_N. The resultant of these two vectors is $F_g sin\theta$. Since the natural motion is down the slope set that direction as +. In some problems it is useful to reverse this (if the object is going up hill). $\boxed{\sum F_{\parallel} = F_g \sin\theta - F_{retarding}}$ $F_N = mg\cos\theta$	Work and energy can work parallel, in the *x*, and in the *y* $W = F_{\parallel} \cdot d_{\parallel}$ Force and distance vectors form similar triangles. $W = F_x \cdot d_x$ Work depends on *F* and *d* being parallel. $W = F_y \cdot d_y$ So any pair of parallel vector will solve the problem $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ $\boxed{W = \Delta U_g = mg\Delta h}$ $mg\Delta h = \frac{1}{2}m(\Delta v)^2$ $\boxed{mgh_i + \frac{1}{2}mv^2{}_i = mgh_f + \frac{1}{2}mv^2{}_f}$
Down a curve  	Kinematics Fail The net force is changing as the vectors F_g and F_N change. In addition the direction is changing. Acceleration is changing. The Kinematic Equations are designed for changing velocity, but only work for uniform (constant) acceleration.	Force Fails The net force is changing as the vectors F_g and F_N change.	Energy is directionless. $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ $\boxed{W = \Delta U_g = mg\Delta h}$ $mg\Delta h = \frac{1}{2}m(\Delta v)^2$ $\boxed{mgh_i + \frac{1}{2}mv^2{}_i = mgh_f + \frac{1}{2}mv^2{}_f}$ A very important case. $\boxed{mgh_{top} = \frac{1}{2}mv^2{}_{bottom}}$ No initial velocity moving to a height of zero.
Pendulum or Swing  $h = R - y$ $y = \sqrt{R^2 - x^2} = R\cos\theta$	Kinematics Fail See accelerating down a curve above. The velocity is zero at either end. The velocity is greatest at the lowest point	Force Fails See accelerating down a curve above. The restoring force is greatest at the ends, as is the acceleration. The restoring force is zero in the middle, and so is the acceleration.	Energy is directionless. $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ $\boxed{W = \Delta U_g = mg\Delta h}$ $\boxed{mgh_{top} = \frac{1}{2}mv^2{}_{bottom}}$

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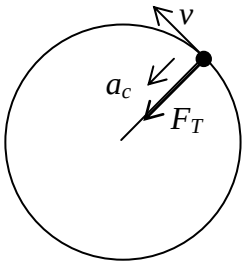
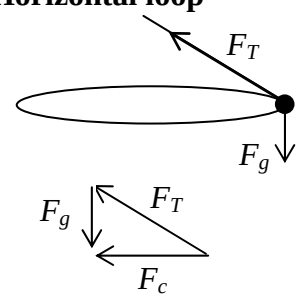
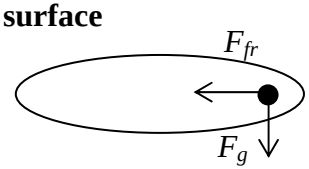
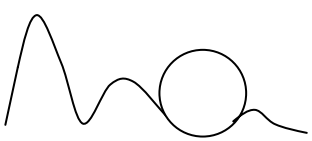
Situation	Kinematics	Force	Energy
Object on string, Vertical loop  	Tangential Velocity Instantaneous velocity is tangent to the circlular motion. $v = \frac{2\pi r}{T}$ Acceleration is toward center, centripetal. $a_c = \frac{v^2}{r}$	Center seeking. Force is centripetal, F_c, is the sum of force in circular motion. Toward center is +. Find tension at the top. $F_c = F_T + F_g$ Find tension at the bottom. $F_c = F_T - F_g$	Unlike previous scenarios, the object definitely has velocity at the top. There is a height difference from top to bottom, but the object has speed at the top as well. And the bottom may not necessarily be the lowest point in the problem $mgh_{top_i} + \frac{1}{2}mv_{top}{}^2{}_i = mgh_{bottom\,f} + \frac{1}{2}mv_{bottom}{}^2{}_f$
Object on string, Horizontal loop  	Tangential Velocity Instantaneous velocity is tangent to the circlular motion. $v = \frac{2\pi r}{T}$ Acceleration is toward center, centripetal. $a_c = \frac{v^2}{r}$	Center seeking. Force is centripetal, F_c, is the sum of force in circular motion. Toward center is +. Find F_c by adding vectors (tip to tail). Then solve for F_T. $F_T = \sqrt{F_c{}^2 + F_g{}^2}$	No work is done $W = F \cdot d \cos\theta$ Where θ is the angle between F and d vectors. $W = F \cdot d \cos 90^o$ $W = 0$ At any instant the direction of motion (tangent to the circle) is perpendicular to the center seeking F_c, the F_T, and the F_g. And for one revolution there is no total displacement from the origin, since a single revolution brings you back to the starting point.
Object turning on flat surface  	Tangential Velocity $v = \frac{2\pi r}{T}$ $a_c = \frac{v^2}{r}$	Why is there circular motion? Object not sliding off disk, or car turning on a road. $F_c = F_{fr}$	No work is done See above.
Roller Coaster 	Need uniform slope Kinematics only work on sections that have constant slope. If the track is curved try energy.	Force centripetal in the loop. To find the speed needed to have passengers feel weightless at the top of the loop $F_c = F_g$	Energy works everywhere, with its directionless advantage. You can solve for any point A using any other point B. Use the complete equation. The car will usually have both speed and height at every point. An exception is the lowest point on the track, or if it starts with zero velocity at the top of a hill (this is unlikely since roller-coasters don't stop at the top of each hill). If it has velocity and height at the top you need to include both. $mgh_A + \frac{1}{2}mv_A{}^2 = mgh_B + \frac{1}{2}mv_B{}^2$

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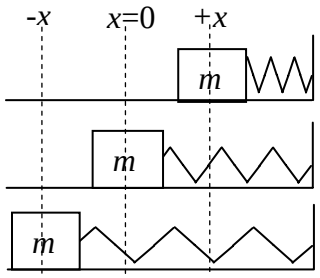
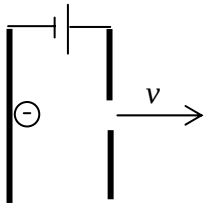
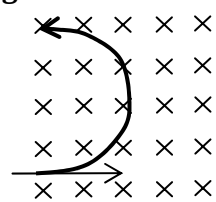
Situation	Kinematics	Force	Energy
Spring  	Kinematics Fail See Down a curve, and Pendulum above. The velocity is zero at maximum +/- x (amplitude) The velocity is greatest at x = 0	Force Fails See Down a curve, and Pendulum above. The restoring force is greatest at maximum +/- x (amplitude). The restoring force is zero at x = 0, and so is the acceleration.	Energy is directionless. $\boxed{W = \Delta K = \frac{1}{2}m(\Delta v)^2}$ $\boxed{W = \Delta U_S = \frac{1}{2}k(\Delta x)^2}$ If it starts at maximum x (amplitude) and it converts all the springs energy into speed of the object pushed / pulled by the spring $\boxed{\frac{1}{2}kx^2 = \frac{1}{2}mv^2}$ $\frac{1}{2}kx^2{}_i + \frac{1}{2}mv^2{}_i = \frac{1}{2}kx^2{}_f + \frac{1}{2}mv^2{}_f$
Particle accelerated by electric field  	Potential difference. Velocity increases. Positive charges go in the opposite direction. But, + particles are more massive, don't accelerate as quickly, and have lower final velocities.	The force of the electric field $\boxed{F = qE}$	Electromagnetism: New forms of energy, but energy is still conserved. $W = \Delta Energy$ $\boxed{W = \Delta K = \frac{1}{2}mv^2}$ $\boxed{W = \Delta U_E = qV}$ $\boxed{qV = \frac{1}{2}mv^2}$ $qV_i + \frac{1}{2}mv^2{}_i = qV_f + \frac{1}{2}mv^2{}_f$
Charged particle parallel to plates 	It acts like a projectile $\boxed{x = v_{xo}t}$ $\boxed{y = \frac{1}{2}at^2}$ get a from F	Electric field is perpendicular Particle is forced toward the plate with opposite sign. $E = \frac{F}{q}$ $\boxed{F = qE}$ $\boxed{\sum F = ma}$ $ma = qE$	Work is done in the direction of the electric field. $W = F \cdot d$ $W = qE \cdot d$
Charged particle in a magnetic field 	Path curved by field. If the field is large enough the particle will follow a circular path. $\boxed{v = \frac{2\pi r}{T}}$ $\boxed{a_c = \frac{v^2}{r}}$	Forced to center by field. $\boxed{F_c = F_B}$ $\boxed{m\frac{v^2}{r} = qvB}$	No work is done $W = F \cdot d \cos 90^o$ $\boxed{W = 0}$ At any instant the direction of motion (tangent to the circle) is perpendicular to the center seeking F_c, and the F_B.

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