# 3–23 Inductance, *RL*, and *LC* Circuits

**Note: Inductance is needed for AP Physics C: physical science majors, calculus based.**

**Self Induction:** When a circuit has a time varying current, an induced *emf* is produced. This induced emf is in the opposite direction of the original current that created it. Consider a simple (loop) circuit consisting of a battery, resistor, and a switch. When the switch is thrown current begins to move. This creates a magnetic field around the wires. The circuit is closed and acts like a single turn loop. The magnetic field induces an *emf* (a second current) in the loop. Due to Lenz's Law this second current counters the original current (reverse direction). The induced current acts to slow the original current that created it. This kind of induced *emf* is known as **self-induced *emf*** or **back *emf***, $\varepsilon_L$.

Inductor: A device intentionally put in a circuit to generate a back emf. A coil of wire generates a stronger magnetic field than a plain wire. As a result, a solenoid used in a circuit acts as an inductor. Using Faraday's Law the formula for inductance is $\varepsilon_L = -N\dfrac{d\phi_m}{dt} = \boxed{-L\dfrac{dI}{dt}}$. $L$ is the inductance (a constant for a specific coil) measured in Henry's (H). The minus sign indicates that the induced *emf* is in the reverse direction of the original current.

***RL* Circuits:** A circuit composed of resistors and inductors. We will assume that the wires in the circuit have negligible resistance and inductance. ***Since the back emf opposes the original current, inductor in circuits oppose changes in current in the circuit.*** If the current wants to rise quickly, an inductor slows the currents rise. If a current wants to stop quickly, an inductor keeps the current going longer.

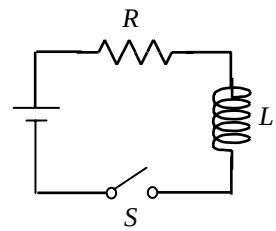

Fig 23.1

Using Kirchhoff's 2nd Law sum all the potential differences of the circuit to zero, $\varepsilon - V_R - \varepsilon_L = 0$. $\varepsilon$ is the *emf* (potential) of the battery, $V_R$ is the potential used by the resistor and $\varepsilon_L$ is the potential used by the inductor. Substitute, $\varepsilon - IR - L\dfrac{dI}{dt} = 0$. Use this formula to analyze the conditions in the circuit, Fig 23.1, after the switch is thrown. Once charge begins to flow as a current the inductor generates a magnetic field. This magnetic field creates a back *emf*, which opposes and slows the current. The more current through the inductor, the greater the back *emf*, and consequently the more drastically the current is slowed.

**Instantaneous Conditions, Right When Switch is Thrown (time $t_0$):** Charges attempt to flow very quickly. Inductors slow this change in current. It is almost as though all the emf is used by the inductor, with no current going to the resistor. (This is the opposite of an RC Circuit, where the current initially goes to the resistor and the capacitor receives none.) Plugging into kirchhoff's 2nd Law, $\varepsilon - (0)R - L\dfrac{dI}{dt} = 0$, and simplifying leads to $\varepsilon = L\dfrac{dI}{dt}$. Since $\varepsilon_L = -L\dfrac{dI}{dt}$, then $\boxed{\varepsilon = \varepsilon_L}$ for just the instant that the switch is thrown.

**At Some Time *t* Later:** Current at a time t is $\boxed{I_t = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$, where the time constant is equal to $\boxed{\tau = \dfrac{L}{R}}$.

**After a very long time:** Inductors only slow the changes in the circuit, they cannot prevent them. Eventually the current reaches a value that is limited by the resistor in the circuit. Plugging into kirchhoff's 2nd Law, $\varepsilon - IR - (0) = 0$, and simplifying leads to $\varepsilon = IR$. So the current in the circuit after a long time is $\boxed{I = \dfrac{\varepsilon}{R}}$.

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**Current vs. Time Graph:** Fig 23.2 shows the current in the circuit over time. Initially the current flows as though the resistor is not present. The inductor slows the rate of change in the current. Instead of rising to its final value instantaneously the current follows this the equation, $\boxed{I_t = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)}$. Eventually the current reaches its full value, which is governed by the resistor alone, $\boxed{I = \frac{\varepsilon}{R}}$. At this point it is as though the inductor does not exist. When the time constant is reached the current is at 63.2% of its maximum value.

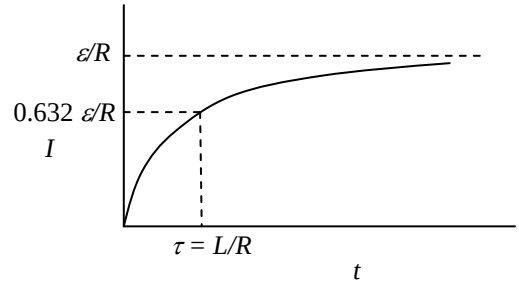

Fig 23.2

**Current That is Trying to Stop:** Fig 23.3 show a circuit with an additional switch. When switch 1 is closed it operates as above. Current attempts to flow quickly through the circuit, but is slowed in its start up by the inductor. In this case we will allow the current has hit its final maximum value, and then we will throw switch 1 off and switch 2 on. This disconnects the battery. The current should die instantaneously, however inductors slow the change in current. Therefore the current will drop slowly in this scenario. This time the following formula applies, $\boxed{I_t = \frac{\varepsilon}{R}e^{-t/\tau}}$ or $\boxed{I_t = I_0 e^{-t/\tau}}$. The inductor acts to oppose the decrease in current. The graph of this scenario is shown in Fig. 23.4.

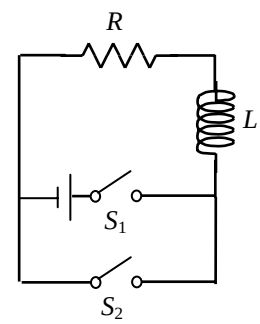

Fig 23.3

**Energy in a Magnetic Field:** Since the inductor opposes the generation of a current, the battery must do work to create a current. Some of the energy goes into internal energy in the resistor (it gets hotter), but the rest is stored in the magnetic field of the inductor. $\boxed{P = IV}$. Adapt Kirchhoff's 2nd Law, $\varepsilon - IR - L\frac{dI}{dt} = 0$, by multiplying the potentials in it by current to change them into power, $I\varepsilon - I^2R - LI\frac{dI}{dt} = 0$.

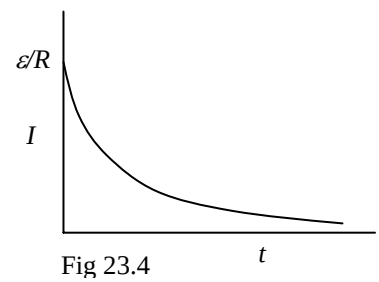

Fig 23.4

Rearrange, $I\varepsilon = I^2R + LI\frac{dI}{dt}$. If we converted these all these terms from power to energy, by multiplying by time, we would have a conservation of energy expression. The energy contained in an inductor is given by the formula $\boxed{U_L = \frac{1}{2}LI^2}$. This is a potential energy, as it is stored and has the potential to do work.

**Oscillations in an *LC* Circuit:** *LC* circuits are composed of inductors and capacitors. The circuit in Fig 23.5 shows the simplest case. In this scenario the capacitor will be assumed to have a full charge. The switch is thrown and the capacitor attempts to discharge quickly. It acts as a source of *emf,* sending a current through the wires. But the inductor acts to slow the discharge. As the capacitor discharges it looses energy. Where does this energy go? To the inductor. Through conservation of energy $U_C = U_L$ and thus $\boxed{\frac{1}{2}QV^2 = \frac{1}{2}LI^2}$ or $\boxed{\frac{1}{2}CV = \frac{1}{2}LI^2}$. The energy is now stored in a magnetic field around the inductor. The magnetic field then collapses, as there is no current to sustain it. This results in a change in flux in the inductor, which generates a current in the wires. But, this collapse is the opposite direction of the original rising magnetic field from the capacitors discharge. Therefore, the direction of this new induced current is the opposite of the original current created by the capacitor. Now the capacitor begins to charge, but since the current is the reverse, the polarity of the plates is switched. If the top plate were positive at the very start of this scenario, then the top plate is now negative. Once the capacitor is fully charges and all the energy from the inductor has passed back to the capacitor the process begins again, and again… (It is exactly like a spring that oscillated above and below the equilibrium position. Sometimes it is +x and sometime -x. The energy just keeps changing from potential to kinetic.)

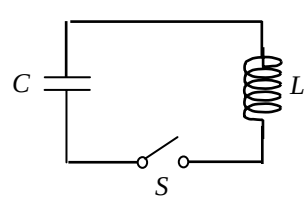

Fig 23.5