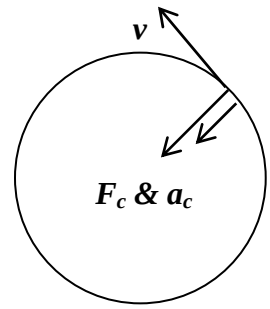
1–9 Uniform Circular Motion and Gravity

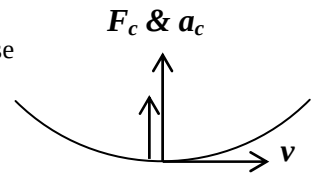
Frequency: How often a repeating event happens. Measured in revolutions per second.

Period: The time for one revolution. $\boxed{T = \frac{1}{f}}$ Time is in the numerator.



Velocity: Direction and thus velocity are continuously changing in circular motion. The magnitude of velocity and speed are not. You can measure an instantaneous velocity, which is tangential to the curve. Tangential Velocity or speed $\boxed{v = \frac{2\pi r}{T}}$

Centripetal Acceleration: Inertia would make a mass leave the circle following the tangential velocity. Instead the direction of the mass is being changed toward the center. In other words the mass is accelerated toward the center. Centripetal means center seeking. $\boxed{a_c = \frac{v^2}{r}}$



Centripetal Force: If an object is changing direction (accelerating) it must be doing so because a force is acting. Remember objects follow inertia (in this case the tangential velocity) unless acted upon by an external force. If the object is changing direction to the center of the circle it must be forced that way. $\boxed{F_c = ma_c}$ $\boxed{F_c = m\frac{v^2}{r}}$

Problem Solving Strategy

1. **As always, ask what the object is doing.** If it is moving in a circle, or even part of a circle, shown above right.
2. **Draw a FBD.** Remember ***F_c is the sum of force*** for circular motion. ***The sum of force is not shown in the FBD***.
3. **Set the direction of motion as positive.** Toward the center is positive, since this is the desired outcome.
4. **Identify the sum of force equation.** In circular motion F_c is the sum of force. F_c can be any of the previous forces.
5. **Substitute the relevant force equations and solve.**

Example 9-1: Vertical Circular Motion

A ball at the end of a string is swung in a vertical circle. Any forces pointing to the center are positive, while force vectors pointing away from the center are negative. Sum the forces. In circular motion F_c is the sum of force. **Find the tension in the string when the ball is at the top and at the bottom.**

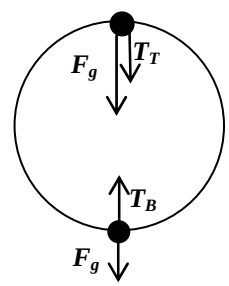


Fig 9.1

$F_c = F_g + T_T$ $\qquad$ $F_c = -F_g + T_B$

$T_T = F_c - F_g$ $\qquad$ $T_B = F_c + F_g$

$\boxed{T_T = m\frac{v^2}{r} - mg}$ $\qquad$ $\boxed{T_B = m\frac{v^2}{r} + mg}$

Example 9-2: Horizontal Circular Motion

A penny on a circular disk rotating horizontally (or a car turning a corner). Something must be keeping it going in a circle. Friction keeps it in place. If friction let go the penny would move due to inertia in a direction tangent to the disk. Force centripetal is the sum of forces for circular motion. **Find a formula for the maximum velocity if the coefficient of friction is known.**

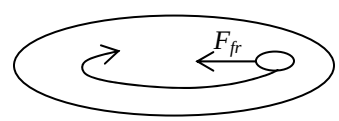


Fig 9.2

$F_c = F_{fr}$ $\qquad$ $\cancel{m}\frac{v^2}{r} = \mu \cancel{m} g$ $\qquad$ $\boxed{v = \sqrt{\mu g r}}$

Example 9-3: Top of a Loop and Apparent Weightlessness

Apparently weightless means that you are in freefall. The only force acting on you is F_g. To feel weightless at the top of the loop the roller coaster car can have no F_N (no pressure from the track). So for an instant at the top the car is not touching the track. **What at the top of the loop makes this possible?**

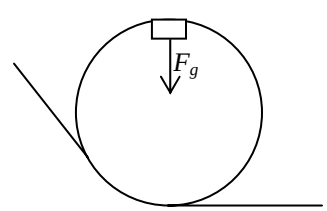


Fig 9.3

$F_c = F_g$ $\qquad$ $\cancel{m}\frac{v^2}{r} = \cancel{m} g$

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Example 9-4: Conical Pendulum

m_1 is suspended by a string that passes through a tube. At the other end of the tube m_2 is hanging from the same string. m_1 is spun at a velocity that keeps m_2 stationary.

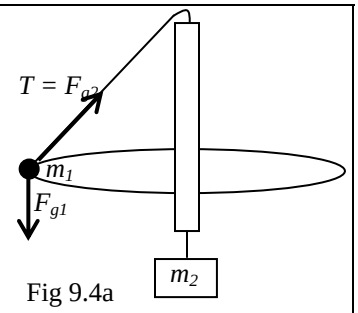


Fig 9.4a

Solve for the force centripetal. Force centripetal is the sum of force that points to the center of the circular motion. The two acting forces on m_1 are causing the circular motion, and they must sum together as force centripetal. If you add the two acting force vectors tip to tail they form a force vector triangle, shown in Fig 9.4b.

$$T^2 = F_c^{\ 2} + F_{g1}^{\ 2} \qquad \boxed{F_c = \sqrt{T^2 - F_{g1}^{\ 2}}}$$

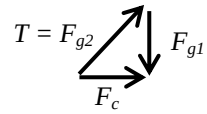


Fig 9.4b

Solve for the tension in the rope. Both masses hang from the rope, so either one can be used. Pick the easiest, in this case the vertically hanging mass. It's FBD is shown in Fig 9.4c. $\sum F = T - F_{g2} \qquad 0 = T - F_{g2} \qquad \boxed{T = F_{g2}}$

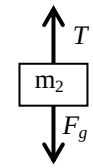


Fig 9.4c

$$F_c = \sqrt{T^2 - F_{g1}^{\ 2}} \qquad m\frac{v^2}{r} = \sqrt{F_{g2}^{\ 2} - F_{g1}^{\ 2}} \qquad \boxed{v = \sqrt{\frac{r\sqrt{F_{g2}^{\ 2} - F_{g1}^{\ 2}}}{m}}}$$

Example 9-5: Gravitron

This is the ride at amusement parks where it spins and the floor drops down, leaving the occupants stuck to the wall.

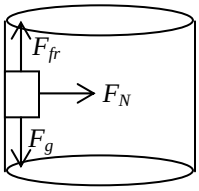


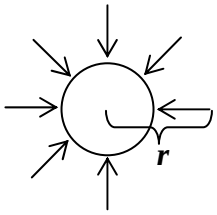
Fig 9.5

Solve for the tangential velocity. You feel pressed against the wall because the wall exerts a normal force toward the center. In other words the normal force is force centripetal, $F_c = F_N$. In the vertical dimension you are prevented from sliding down the wall by an upward and equal friction force, $F_{fr} = F_g$. Friction depends on force normal. $F_{fr} = \mu F_N$. Put all these equations together, and substituting for F_c and F_g.

$$F_c = F_N \qquad F_c = \frac{F_{fr}}{\mu} \qquad F_c = \frac{F_g}{\mu} \qquad m\frac{v^2}{r} = \frac{F_g}{\mu} \qquad \cancel{m}\frac{v^2}{r} = \frac{\cancel{m}g}{\mu} \qquad \boxed{v = \sqrt{\frac{rg}{\mu}}}$$

Gravity: One of the fundamental forces. This force is a field force, and the field is g, the acceleration of gravity. Every mass in the universe generates a gravity field. The gravity field is directed toward the center of mass. While the nature of the force is not understood the mathematics are detailed in Newton's Law of Universal Gravitation $\boxed{F_g = G\frac{m_1 m_2}{r^2}}$ where $G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$. This equation is the force between two masses. Remember the force between objects is equal and opposite. Combine this with the equation for weight $\boxed{F_g = mg}$ to get $mg = G\frac{m_1 m_2}{r^2}$, which simplifies as $\boxed{g = G\frac{m}{r^2}}$.

Each equation has its usefulness depending on the situation. The last equation is important for finding the gravity field value g around any mass at a distance r. To find the gravity at a point in space near Earth, use the mass of earth (which creates the gravity) and the distance from Earth's center. r is not a radius, but is the distance measured from center of a mass. r is used since gravity radiates in rays from the center of mass in a spoke like manner. Viewed in this way every distance in gravity is a radius. If you calculate g at a point in space near a mass you also know g for all points on a sphere of that radius (equipotential, since all points have the same potential energy).



Inverse Square Law: This can be used for both formulas with r^2 in the denominator. If r doubles (x2), invert to get ½ and then square it to get ¼. Gravity is ¼ its original value so F_g is ¼ of what it was and g is ¼ of what it was. Multiply the old F_g by ¼ to get the new weight, or multiply g by ¼ to get the new acceleration of gravity.

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Example 9-6: Gravity on an Unknown Planet

Mars has roughly half the radius of Earth and has one-tenth the mass.

What is the gravity on the surface of Mars? Many students want to look up the radius and mass of Mars and plug into this equation $\boxed{g = G\frac{m}{r^2}}$. But there is another way. The problem gives us the relationship to Earth for a reason. We also already know the gravity on Earth $9.8 = G\frac{m_{Earth}}{r_{Earth}^{\ 2}}$. What if we just use some logic and the Inverse Square Law? The gravity on Mars is going to be Earth's gravity adjusted by pretending Earth shrinks to half its radius and one-tenth its mass.

$g_{Mars} = G\frac{(m_{Earth}\times 0.1)}{(r_{Earth}\times 0.5)^2}$ $\quad$ $g_{Mars} = G\frac{m_{Earth}}{r_{Earth}^{\ 2}}\times\frac{(0.1)}{(0.5)^2}$ and we already know that $9.8 = G\frac{m_{Earth}}{r_{Earth}^{\ 2}}$, so

$g_{Mars} = 9.8\times\frac{(0.1)}{(0.5)^2}$ $\quad$ $\boxed{g_{Mars} = 3.92\, m/s^2}$. Again, just pretend Earth shrinks to become Mars. This last line is all the work you need to show.

Example 9-7: Superposition of Gravity Fields

Superposition is a term referring to the addition, or superimposing, of two or more force fields. In Fig 9.7a mass A and mass B both create gravity at all points in space to infinity. If an object is positioned at point P it will feel the gravity of both masses. The two gravities must be added together using vector addition. $m_A = 2.00\times10^{20}$ kg, $m_B = 4\times10^{20}$ kg, The masses are 2.00×10^8 m apart. Point P is located at a point half way between the masses.

What gravity is felt at point P? First solve for the gravity of each planet, at point P, separately. Use $g = G\frac{m}{r^2}$

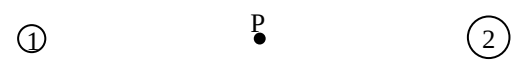

Fig 9.7a

$$g = (6.67\times10^{-11})\frac{(2.00\times10^{20})}{(1.00\times10^8)^2} = \boxed{1.33\times10^{-6}\, m/s^2} \text{ toward } m_1 \text{ (left)}$$

$$g = (6.67\times10^{-11})\frac{(4.00\times10^{20})}{(1.00\times10^8)^2} = \boxed{2.67\times10^{-6}\, m/s^2} \text{ toward } m_2 \text{ (right)}$$

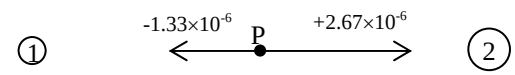

Fig 9.7b

g is a vector, and vectors have direction. Assign a positive sign to the vector pointing right and a negative sign to the vector pointing left, as shown in Fig 9.7b. Then add the two vectors together $(-1.33\times10^{-6}) + (+2.67\times10^{-6}) = \boxed{+1.33\times10^{-6}}$. The positive answer implies that the gravity at point P is 1.33×10^{-6} m/s^2 directed toward the right.

If a 100 kg mass were to be positioned at point P, what would the force of gravity be? The beauty of finding g is that you can easily apply it to any mass at that location to find the force of gravity $F_g = (100)(1.33\times10^{-6}) = \boxed{1.33\times10^{-4}\, N}$

Potential Energy Revisited: There is another equation to find potential energy using the universal gravity constant. Use the work formula and work energy theorem, $W_g = \Delta U_g = F_g \Delta r$. Set the initial displacement as zero and it simplifies to $U_g = F_g r$. Use this with $\boxed{F_g = -G\frac{m_1 m_2}{r^2}}$ to get $\frac{U_g}{r} = -G\frac{m_1 m_2}{r^2}$. This simplifies to $\boxed{U_g = -G\frac{m_1 m_2}{r}}$.

Where did the minus sign come from? Suddenly it is added to Newton's Law of Universal Gravitation. This is the formal version of the law. It can be used with either a positive sign (simplified and common version) or a negative sign (formal version) and is situational dependent. In formal physics a point at infinity is said to have zero potential energy. Since a central point of zero potential energy cannot be located in the universe, it makes sense to pick infinity to be zero potential energy. All points in the universe are the same infinite distance from infinity. However, this means that close to Earth's surface potential energy is negative. It is common practice when viewing planets from a great distance to set infinity as $U_g = 0$, and when on a planets surface to set the lowest height as $U_g = 0$. These are just conventions used to make specific problems easier to solve. Remember, the exact energy that an object has is not really important. What matters is how much of that energy is usable to do work. And work is a change in energy. Therefore, we can really declare any point as zero energy and measure changes from that point.

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