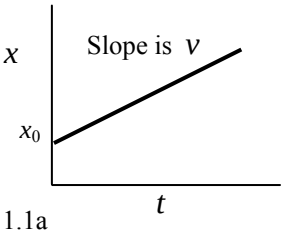
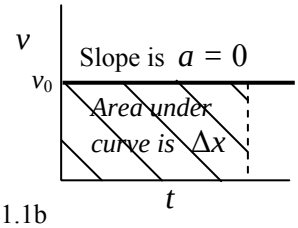
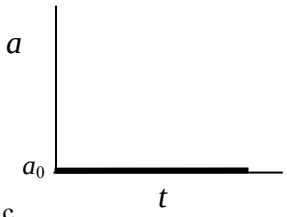
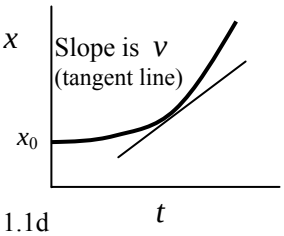
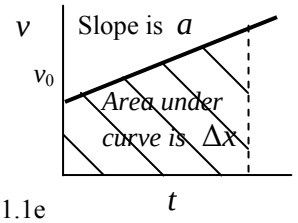
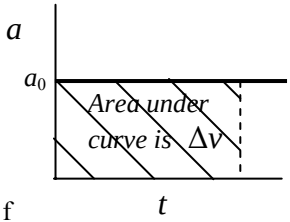
Graphing

	Displacement–Time Graph	*Velocity–Time Graph*	*Acceleration–Time Graph*
Constant Velocity	 Fig 1.1a	 Fig 1.1b	 Fig 1.1c
Uniform Acceleration	 Fig 1.1d	 Fig 1.1e	 Fig 1.1f

Slopes of Curves are an important analytical tool used in physics. Any equation that can be manipulated into the format $y = mx + b$ can be represented and analyzed graphically. As an example: $v = v_0 + at$ can be rearranged slightly into $v = at + v_0$. Compare this equation to the equation of a line. It is apparent that a is the slope and that v_0 is the y-intercept (Fig 1.1b and Fig 1.1e). What equation generates velocity in Fig. 1.1a and Fig 1.1d?

Area Under a Curve is another important graphical tool. Multiply the y-axis (height) by the x-axis (base) and determine if this matches any known equations. For example: Figures 1.1b and 1.1e are velocity-time plots. Simply multiply $v \times t$. This is a rearranged form of the equation $v = x/t$. The form obtained from the graph is $x = vt$, which means that displacement is the area under the velocity-time plot.

Calculus: ***Required in AP Physics C*** (optional for AP Physics B students)***. Calculus is taught in math class. These review sheets will focus on the Physics aspect of solutions. Calculus steps may not be shown. Solution will be up to the student.***

The equations outlined in the previous page work well in the following situations.

- Linear Functions: involving constant velocity and acceleration, as diagrammed in the above graphs (except Fig 1.1d).
- Nonlinear Functions: scenarios where the problem is seeking information about a change in a quantity, Δx or Δv.
- Nonlinear Functions: scenarios where the problem is seeking an average velocity in an interval.

Calculus is needed to find the slopes of nonlinear functions and the areas under nonlinear curves.

1. **Velocity: Slope of the displacement-time curve.**

   $v = \dfrac{dx}{dt}$ example: $v = \dfrac{d}{dt}\left(x_0 + v_0 t + \dfrac{1}{2}at^2\right) = v_0 + at$ $\qquad v = v_0 + at$

2. **Acceleration: Slope of the velocity-time curve.**

   $a = \dfrac{dv}{dt}$ example: $a = \dfrac{d}{dt}\left(v_0 + at\right) = a$

3. **Velocity: Area under acceleration-time curve.** (Note: if $c = v_0$ cannot be found, then you can only solve for Δv)

   $v = \int a\ dt$ example: $v = \int (a)\ dt = c + at$ $\qquad v = v_0 + at$

4. **Displacement: Area under the velocity-time curve.** (Note: if $c = x_0$ cannot be found, then you can only solve Δx)

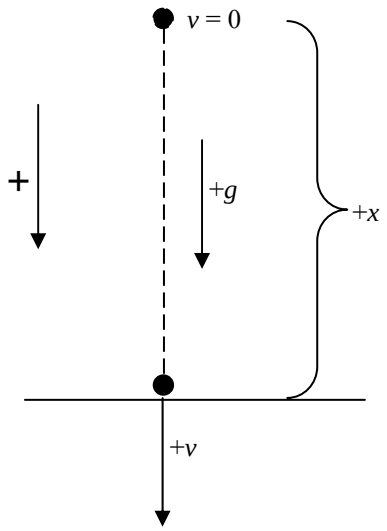
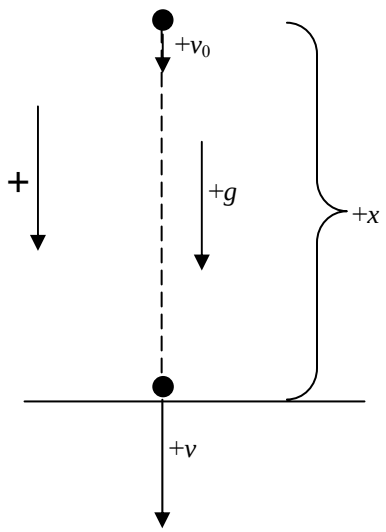
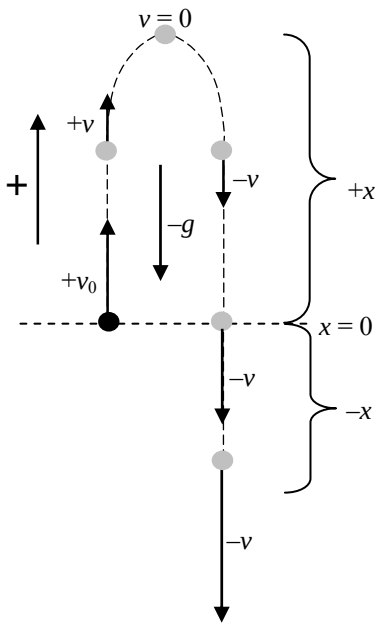
   $x = \int v\ dt$ example: $x = \int \left(v_0 + at\right)\ dt = c + v_0 t + \dfrac{1}{2}at^2$ $\qquad x = x_0 + v_0 t + \dfrac{1}{2}at^2$

 © R H Jansen

Falling Bodies: Objects moving vertically under the influence of gravity. Earth's surface gravity is $g = -9.8$ m/s^2. This speeds objects up (+ acceleration), but is directed downward (–). Objects can be thrown up, down, or just be dropped. They can land below, at the same height, or above the origin. They come to an instantaneous stop at the highest point in flight.

$$\boxed{v_y = v_{0y} + gt} \qquad \boxed{y = y_0 + v_{0y}t + \frac{1}{2}gt^2} \qquad \boxed{v_y^{\,2} = v_{0y}^{\,2} + 2g\left(y - y_0\right)}$$

The positive and negative signs can cause trouble in these problems. The easiest way to handle the signs is set the direction of initial motion as positive and then to ensure all signs are consistent with this decision. This has one huge benefit. It eliminated the double sign on acceleration. When initial velocity v_0 is used to anchor direction, then a positive acceleration means speeding up and negative acceleration involves slowing down.

Dropped from rest	**Thrown downward**	**Thrown upward**
$v_0 = 0$, but it will move down initially	v_0 is directed downward	v_0 is directed upward
Set downward as positive direction	Set downward as positive direction	Set upward as positive direction
		
Everything is positive and easy	Everything is positive and easy	This is difficult, and depends on where in the flight the problem ends.

Kinematics Problems Involving Changes in the Magnitude of Acceleration

If the magnitude of acceleration changes while solving a kinematics problem then the problem must be solved in separate parts. Unlike displacement x and velocity v, the Kinematic equations do not contain the variables a_0 and a.

Example 1-3: More than one acceleration

A car initially at rest accelerates at 4 m/s^2 while covering a distance of 100 m. Then the car continues at constant velocity for 500 m. Finally it slows to a stop with a deceleration of 3 m/s^2. Determine the total time of this displacement.

Acceleration Phase: $x = \frac{1}{2}at^2$ $\qquad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(100)}{(4)}} = 7.07\,s$ and $v = v_0 + at = (0) + (4)(7.07) = 28.3\,m/s$

Constant Velocity Phase: $v = \frac{\Delta x}{t}$ $\qquad t = \frac{\Delta x}{v} = \frac{(500)}{(28.3)} = 17.7\,s$ $\qquad t = \frac{500}{v}$

Deceleration Phase: $v = v_0 + at$ $\qquad t = \frac{v - v_0}{a} = \frac{(0) - (28.3)}{(-3)} = 9.43\,s$

Total Time = 7.07 + 17.7 + 9.43 = 34.2 s

 © R H Jansen