1–6 Work, Energy, and Power

Work: Force applied to an object that moves a distance.

$\boxed{W = \mathbf{F} \bullet \Delta \mathbf{r} = F \Delta r \cos\theta}$ θ is the angle between direction of motion and applied force.

Work is a Scalar (Dot) Product: The dot product (**A**·**B**) of two vectors **A** and **B** is a scalar and is equal to $AB\cos\theta$. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^o$ (parallel) and when $\theta = 180^o$ (anti-parallel). The scalar is zero when $\theta = 90^o$ (perpendicular).

Work can be solved with either version of the formula. We will use both in example 6–1 below. The formula $W = \mathbf{F} \bullet \Delta \mathbf{r}$ involves vectors, which are annotated in bold print. Vectors have both magnitude and direction. The positive and negative values of force and displacement are important when using this version of the formula. The other version $W = F \Delta r \cos\theta$ involves italicized print indicating that only the magnitude of each vector is needed. Only positive numbers are used. The angle in the formula is the angle measured between the two vectors. In this version of the formula $\cos\theta$ solves the directional aspect. Compare both methods in the example below.

Example 6-1: Various Orientations of Force and Displacement.

In the first three scenarios a force $F = 5$ N acts on a mass which is displaced $r = 2$ m.

1. Force vector is parallel to the displacement vector and points in the same direction:

$W = \mathbf{F} \bullet \Delta \mathbf{r}$ $\qquad$ $W = F \Delta r \cos\theta$

$W = (+5)(2)$ $\qquad$ $W = (5)(2)\cos 0^o$

We decided F was positive $\qquad$ Here the angle solves for the positive

$W = (+5)(2) = \boxed{10\,\text{J}}$ $\qquad$ $W = (5)(2)(+1) = \boxed{10\,\text{J}}$

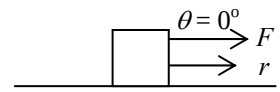

Fig 6.1a

2. Force vector is parallel to the displacement vector and points in the opposite direction:

$W = \mathbf{F} \bullet \Delta \mathbf{r}$ $\qquad$ $W = F \Delta r \cos\theta$

$W = (-5)(2)$ $\qquad$ $W = (5)(2)\cos 180^o$

We decided F is negative $\qquad$ Here the angle solves for the negative

$W = (-5)(2) = \boxed{-10\,\text{J}}$ $\qquad$ $W = (5)(2)(-1) = \boxed{-10\,\text{J}}$

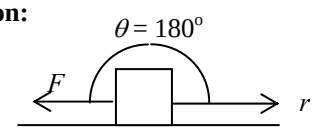

Fig 6.1b

Forces opposing motion are negative, and are associated with negative acceleration and negative work.

3. Force and displacement vectors are perpendicular:

$W = \mathbf{F} \bullet \Delta \mathbf{r}$ $\qquad$ $W = F \Delta r \cos\theta$

$W = (0)(2)$ $\qquad$ $W = (5)(2)\cos 90^o$

We decided F has no affect on motion $\qquad$ Here the angle solves for the zero

$W = (0)(2) = \boxed{0\,\text{J}}$ $\qquad$ $W = (5)(2)(0) = \boxed{0\,\text{J}}$

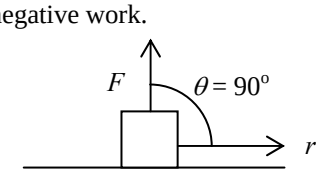

Fig 6.1c

Forces acting perpendicular to motion have no affect in the direction of the original motion. They do not speed up or slow the object in the direction being investigated. The object experiences inertia (stays at rest or continues at constant velocity) in the direction it was originally moving. No work is done in the original direction of motion. The force may accelerate the object in the perpendicular direction. However, this will no affect the direction of motion.

4. Force and displacement vectors are at angles other than parallel or perpendicular:

In this scenario $F = 5$ N at 37° and $r = 2$ m

$W = \mathbf{F} \bullet \Delta \mathbf{r}$ $\qquad$ $W = F \Delta r \cos\theta$

To use this formula you need a component of F parallel to r

$W = F_x r$ $\qquad$ $W = (5)(2)\cos 37^o$

$W = (5\cos 37^o)(2) = \boxed{8\,\text{J}}$ $\qquad$ $W = (5)(2)(0.8) = \boxed{8\,\text{J}}$

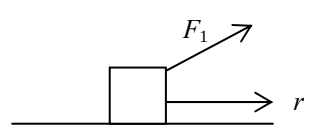

Fig 6.1d

Both methods result in the same calculations. They are just derived with slight differences in problem solving logic. I prefer using the formula on the left. I am in the habit of searching out my own components anyway. I know that any components perpendicular do not matter, since W = 0 in these cases. So I look for components of force that match the displacement. You can also look for components of displacement that match force. I prefer finding the correct component, since the formula on the right requires a specific angle that is not always the given angle in the problem. By solving for components I take control of the problem and avoid plugging in wrong angles.

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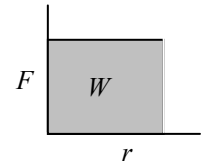
Work is the Area Under the Force Displacement Curve

This is the integral of the force distance function in the calculus based course. In the non-calculus course these areas will be simple enough (squares, rectangles, and triangles) to allow us to use geometry to find the area. AP Physics C students need to be ready to deal with complex curves using the calculus expressions below.



Calculus: $\boxed{W = \int F\ dr}$ If the integral of force over an interval of displacement is work, then it should follow that the derivative of work with respect to the same interval of displacement is force. $F = \dfrac{dW}{dr}$

Energy: A capacity an object has to do work. There are many forms of energy including, mechanical, chemical, electrical, thermal, nuclear, etc. An object may have many types and amounts of energy at the start of a problem. Calculating the total energy an object has is impossible, and it is also unnecessary. During the course of a problem the type and amount of energy, or both, may change, and usually this change is restricted to only a few forms of energy. Energy can also be passed from object to object. Instead of trying to find the total energy we will focus on the forms of energy that change or that transfer from object to object. Tracking the change or movement of energy is very manageable, flexible, and extremely useful.

Mechanical Energy: The sum of Kinetic and Potential (gravitational) Energies

Kinetic Energy: Energy of motion. Depends on mass (inertia of the object) and velocity. Velocity has a greater effect on kinetic energy as it is squared in the formula $\boxed{K = \frac{1}{2}mv^2}$. Double a cars mass and you double kinetic energy. Double a cars velocity and you quadruple kinetic energy.

Potential Energy (gravitational): Energy of position. Depends on an objects mass, the gravity pulling the object, and on the height that the object is located at $\boxed{U_g = mgh}$. For mathematical simplicity and convenience the lowest possible point that the object can reach is designated to have zero height, and h is measured from this point. This is arbitrary and any point can be chosen, but choosing the lowest point as zero avoids dealing with negative heights.

Work Revisited: Think of work as the energy that is added (+***W***) to the system or subtracted (-***W***) from the system.

System: The object that the problem is focusing on.

Environment: The surrounding. The entire universe, except the object in the problem.

Work Energy Theorem: $\boxed{W = \Delta Energy}$ Work put into a system equals the change in energy of the system.

Example 6-2: Work and Potential Energy

Lifting a Mass: In order to lift a mass at constant velocity a force must be directed upward and be equal to the force of gravity. Use $W = F\Delta r\cos\theta$. Substitute F_g for F and height h for r. $W = F_g h\cos\theta$. The force and displacement are in the same direction, so $\theta = 0^o$ and thus $W = F_g h$. Substitution leads to $W = mgh$. Strangely this is the formula for gravitational potential energy? $U_g = mgh$. So does $W = U_g$?

Not really. Work-Energy Theorem states that work is a change in energy.

$W = \Delta Energy \qquad W = \Delta(mgh) \qquad W = mgh_f - mgh_i$

But if we set the lowest height in the problem (we lifted the mass so h_i is the lowest point) to be zero, then $W = mgh_f$.

So, if we arbitrarily identify the lowest height as zero, then the work done to raise an object does equal the energy it has at its new position, relative to its starting point. We are doing positive work, and this adds to the energy of the object.

If we lift an object by doing 20 J of work, then the object will have 20 J of additional energy. ***If we pretend that the object had 0 J at the start of the problem***, then it now has 20 J at the end. Caution: No object ever has zero energy. This is just a mathematical trick or simplification. We are not concerned with the amount of total energy the object has. Instead we are focusing on the change in energy. Many forms of energy are present (chemical, electrical, nuclear, etc.), but they did not change in this problem. The only energy that changed was the energy of height or position, known as potential energy. For convenience we set this energy to zero at the start of the problem (we simply moved the number line). Besides avoiding negative heights it allows easier analysis of useable energy and energy flow. The work done to lift the mass is added to the mass and becomes the final energy of the mass. This is really just the added energy, and since the object cannot fall any

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further than the lowest height (zero potential energy), it can only loose this much potential energy. ***We are really just tracking changes in usable energy.*** This is why Work and Work-Energy Theorem is such a useful entity.

20 J of work were added to the system, doing positive work and raising the systems energy. Where did it come from? It came from the environment. We will soon learn that energy is conserved and just either moves (transfers) between locations (system and environment) or it changes form (potential to kinetic, etc.). If you drop the object it will loose 20 J of potential energy. Since it is loosing energy, 20 J of negative work is done by gravity. Where does this energy go? It turns into 20 J of kinetic energy. Then the object hits the table and stops. Now it has lost all of its height (potential energy) and all of its velocity (kinetic energy). Where did the 20 J go? There was a sound, so molecules of air were pushed aside. The kinetic energy of the molecules increased. The table vibrated. The kinetic energy of the molecules in the table increased. This vibration is felt as heat. Where are the air molecules and table molecules? They are in the environment. So the 20 J returned to the environment. What do you mean by returned to the environment? Well, when I picked up the mass in the first place I was part of the environment, so the original 20 J used to lift the mass came from the environment in the very beginning.

Conservation of Energy: Energy cannot be created or destroyed, but it can change forms.

There are many forms of energy. Those that will be used in this course are shown in the energy wheel below (Note: AP Physics C students will not cover Thermal or Modern). All the forms of energy are included here as this serves as a review of the entire year. If any energy is unfamiliar to you at this point in the year don't worry, we will cover them soon enough.

If energy can change form, then any energy can be equal to any other energy. If you drop an object the object looses height, but gains velocity. In this case potential energy is turning into kinetic energy. If the object looses all of its height, then there is a 100% transfer of energy to kinetic energy. $\boxed{mgh_i = \frac{1}{2}mv_f^{\,2}}$. But, what if the energy is an incomplete transfer. What if the problem ends before reaching zero height (zero potential energy). The object will then have two energies at the end. $\boxed{mgh_i = mgh_f + \frac{1}{2}mv_f^{\,2}}$. Energy is a scalar. It is directionless and simply adds without worrying about direction.

Basically, conservation of energy means that the total energy at the beginning of the problem must equal the total energy at the end of the problem. What if the object has height and is moving at the beginning of the problem, and still has height and is moving at the end of the problem? $\boxed{mgh_i + \frac{1}{2}mv_i^{\,2} = mgh_f + \frac{1}{2}mv_f^{\,2}}$. The total energy in the problem is either $\boxed{E_{total} = mgh_i + \frac{1}{2}mv_i^{\,2}}$ or $\boxed{E_{total} = mgh_f + \frac{1}{2}mv_f^{\,2}}$. The total energy is conserved, so the total energy is present at the beginning, and it is still present at the end. What happens if the object is thrown straight upward, from the lowest point, and then reaches the highest point of flight where the velocity is zero? $\boxed{\frac{1}{2}mv_i^{\,2} = mgh_f}$. What if it is thrown upward at an angle so that when it reaches its highest point it still has some velocity in the x- direction? $\boxed{mgh_i = mgh_f + \frac{1}{2}mv_f^{\,2}}$. We can see that conservation of energy is a very flexible and powerful tool.

Remember: Energy is directionless. Simply ask yourself:

1. What energy/energies are present initially and add them up on the left.
2. What energy/energies are present finally and add them up on the right.

Can energy be lost? No! Lost energy goes to the environment. A car (system) looses energy due to air resistance, so air molecules (environment) gain energy and move faster. Energy is conserved in the universe.

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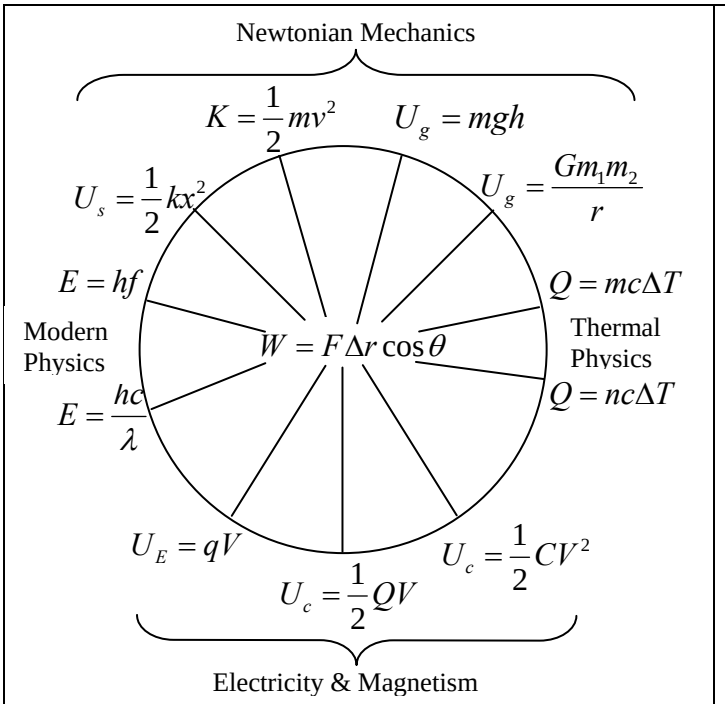

The wheel to the left shows all the energies learned in this class around the outside. They are grouped into the five major strands of physics. In the center is work.

Conservation of Energy: Energy cannot be created or destroyed it can change forms.

Changing forms means that any energy/energies initial around the outside of the circle can change into any other energy/energies final. $\boxed{\sum Energy_i = \sum Energy_f}$

In this class we often assume 100% energy transfer. One energy may be present initially, which disappears, and another energy is present at the end of the problem. In these 100% energy transfer problems any energy around the wheel can equal any other energy. If more than two energies are mentioned then you simply add up all energies present initially and set them equal to the total of all the final energies.

Work Energy Theorem: Work is a change in any energy. $\boxed{W = Any\ Energy_f - Any\ Energy_i}$

Most of the time one of the energies, either final or initial is equal to zero. So then Work can equal any energy around the outside. And since $W = Fr$, then Fr can equal any energy around the outside.

When do we use Conservation of Energy as Opposed to Work Energy Theorem? Conservation of energy is used when energy stays in the system and simply changes form. Work Energy Theorem is used when energy moves from the environment to the system and vice versa. You will see me use a hybrid of the two, in the examples below, when energy is lost to the environment due to friction.

The following is an incomplete list of examples. Many of these are common problems, and have been represented in AP Exams. In addition many are based on 100% efficiency of energy transfer. This is not really the case (see thermodynamics, soon), but it makes the problems easier in the same way that an airless and frictionless world helped us to start kinematics.

Example 6-3: Height Turning Into Velocity

The path does not matter, only the change in height.

For a 100% transfer: $\boxed{mgh_i = \frac{1}{2}mv_f^{\ 2}}$

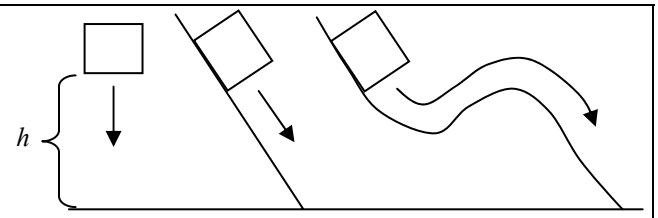

Fig 6.3

Example 6-4: Roller Coaster

A roller coaster is a good example of incomplete transfer of energy. At different points along the track there are various amounts of both kinetic and potential energy. One key to all these height problems, measured from a planetary surface, is to declare the lowest point in a problem to be $h = 0$. This gives a reference that is easy to add or subtract from. Then if you are given the height at any point on the track you can find the carts velocity:

$$\boxed{mgh_i + \frac{1}{2}mv_i^{\ 2} = mgh_f + \frac{1}{2}mv_f^{\ 2}}$$

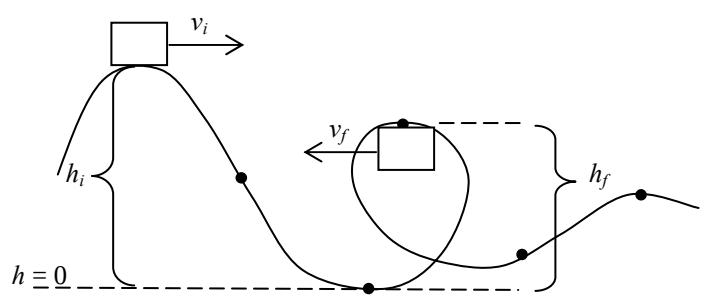

Fig 6.4

Example 6-5: Friction Stops an Object

Fig 6.3a shows an object that had height and stopped due to friction. Friction is a force and the only way to use a force in an energy equation is to multiply the force by the displacement, in other words use work. In this case it is the work of friction.

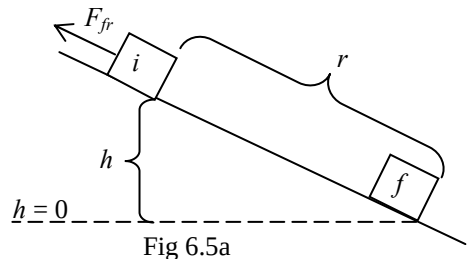

Fig 6.5a

$$mgh_i = W_{fr} \qquad \boxed{mgh_i = F_{fr}r}$$

If the object in Fig 6.3a did not stop, but was just slowed by friction then it would still have some kinetic energy.

$$mgh_i = W_{fr} + \frac{1}{2}mv_f^{\,2} \qquad \boxed{mgh_i = F_{fr}r + \frac{1}{2}mv_f^{\,2}}$$

In figure 6.3b the object is sliding along a horizontal surface. It initially has kinetic energy and is stopped by friction.

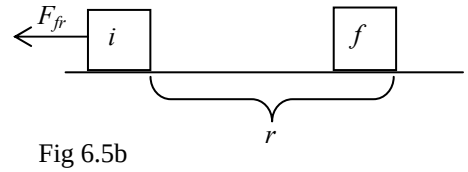

Fig 6.5b

$$\frac{1}{2}mv_i^{\,2} = W_{fr} \qquad \boxed{\frac{1}{2}mv_i^{\,2} = F_{fr}r}$$

If the object in Fig 6.3b did not stop, but was just slowed by friction then it would still have some kinetic energy.

$$\frac{1}{2}mv_i^{\,2} = W_{fr} + \frac{1}{2}mv_f^{\,2} \qquad \boxed{\frac{1}{2}mv_i^{\,2} = F_{fr}r + \frac{1}{2}mv_f^{\,2}}$$

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Power: Power is the rate of work, or rate of energy change. In other words it is the rate that energy is used, transferred, or generated during a one second interval. Since it involves energy, power is by extension as important.

$\boxed{P = \frac{W}{t}}$ you can substitute for work $P = \frac{F\Delta r}{t}$, and if you note that displacement over time is velocity then,

$\boxed{P = Fv}$. The first boxed equation is useful when you have work or energy, the second is useful when you have force. Even though the first equation contains the expression for work, you must be flexible. You must realize that work is a change in any energy $\boxed{P = \frac{\Delta Any\ Energy}{t}}$. You can plug in any energy from the preceding page.

$\boxed{P = \frac{mgh}{t}}$, $\boxed{P = \frac{1/2mv^2}{t}}$, etc. ***<u>If a problem contains any form of energy or work or the units of joules, and any quantity of time or any units of time, then it will involve power.</u>***

If energy is flexible then so is power. Occasionally a question gives power as a variable, but you need energy to solve the problem. Simply set the time equal to one second, then $P = \frac{W}{1s}$ and work/energy will have the same numerical value (but different units) as power for that one second. Use this value for Work/Energy to solve the problem. Just remember that all answers obtained in the problem are based on one second. If time is given later on, just multiple the energy of "one second" by the number of seconds and you've got your answer.

Powerful machines do more work in the same time, or the same work in less time.

Calculus: $\boxed{P = \frac{dW}{dt}}$ Power is another rate (function of time) and is therefore a derivative expression. Integrating power during a time interval will return work or energy values. $\boxed{W = \int P\ dt}$

<u>Solve problems by looking for energy, work, and power first, then force, last of all kinematics.</u>

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