1–2 Vectors and Vectors in Kinematics

Scalar: A physical quantity described by a single number and units. A quantity describing ***magnitude*** only.

Vector: Many of the variables in physics equations are vector quantities. Vectors have ***magnitude*** and ***direction***.

Magnitude: Size or extend. The numerical value.

Direction: Alignment or orientation with respect to set location and system of orientation, such as a coordinate axis.

Notation: $\vec{A}$ or $\xrightarrow{\quad A \quad}$ The length of the arrow represents, and is proportional to, the vectors magnitude. The direction the arrow points indicates the direction of the vector.

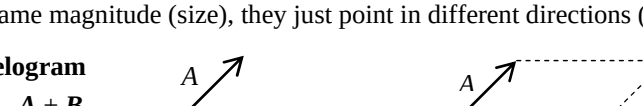
Negative Vectors: Have the same magnitude as their positive counterpart, but point in the opposite direction.

If $\xrightarrow{\quad A \quad}$ then $\xleftarrow{\quad -A \quad}$

Vector Addition and subtraction

Think of it as vector addition only. The result of adding vectors is called the **result**ant $\vec{R}$.

$\vec{A}+\vec{B}=\vec{R}$



When you need to subtract one vector from another think of the one being subtracted as being a negative vector.

$\vec{A}-\vec{B}$ is really $\vec{A}+\left(-\vec{B}\right)=\vec{R}$

A negative vector has the same length as its positive counterpart, but its direction is reversed. ***<u>This is very important</u>***. In physics a negative number does not always mean a smaller number. Mathematically –2 is smaller than +2, but in physics these numbers have the same magnitude (size), they just point in different directions (180° apart).

Vector Addition: Parallelogram

A + ***B***

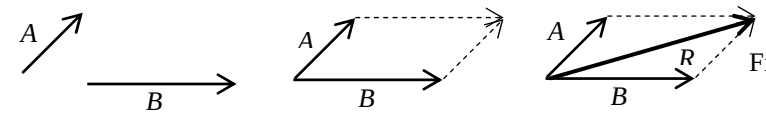


Fig 2.1a

Tip to Tail

A + ***B***

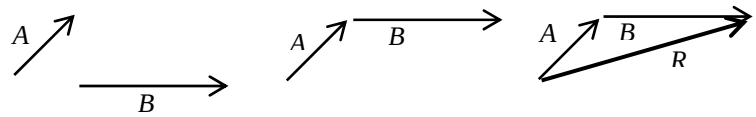


Fig 2.1b

Both methods arrive at the exact same solution since either method is essentially a parallelogram. In some problems one method is advantageous, while in other problems the alternative method is superior.

Reporting Magnitude and Direction:

Component Method: A vector a can be reported by giving the components along the *x*- or *y*-axis. Reporting a vector this way is formally done by employing the unit vectors **i** and **j**. As an example: vector A in fig 2.2a would be $\boxed{A = A_x\mathbf{i} + A_y\mathbf{j}}$, where, $|\mathbf{i}|+|\mathbf{j}|=\mathbf{1}$. There is at third component vector **k** used for three dimensional problems involving the *z*-axis. The vector in Fig. 2.2b shows a numerical application of the component method. In this example you are given the polar coordinates $A = 5\ at\ 37^o$. Using trigonometry the components can be established.

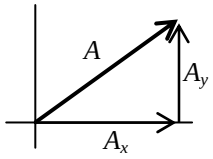


Fig. 2.2a

$\boxed{A_x = A\cos\theta} = 5\cos 37^o = 4$ and $\boxed{A_y = A\sin\theta} = 5\sin 37^o = 3$. Then A can be expressed as follows: $A = 4\mathbf{i} + 3\mathbf{j}$

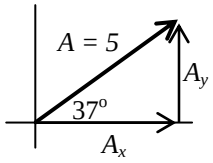


Fig. 2.2b

Polar Coordinates: Vector A in Fig. 2.2b is reported in polar coordinates, $A = 5\ at\ 37^o$. This is simply the length of a vector and its angle measured counterclockwise with respect to the positive *x*-axis. (Negative angles are allowed and indicate that direction was measured clockwise from the +*x*-axis. If the component vectors are given, $A = 4\mathbf{i} + 3\mathbf{j}$, Pythagorean theorem is used to establish the length of the parent vector $\boxed{A = \sqrt{A_x^{\,2} + A_y^{\,2}}} = \sqrt{4^2 + 3^2} = 5$.

Arctangent is used to find the direction $\boxed{\theta = \tan^{-1}\dfrac{A_y}{A_x}} = \tan^{-1}\dfrac{3}{4} = 37^o$. ***But, watch out! The angle arrived at by the arctangent formula may not be the final answer. The quadrant that the final vector lies in must be established, and an adjustment to the angle may be needed in order to provide an answer that extends from the +x-axis.***

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Component Advantage In Vector Addition:

1. If vector components along an axis are used, direction can be specified with + and – symbols. Vectors A and B in Fig 2.3 have been converted into components.

   $A = (+3)\mathbf{i} + (+4)\mathbf{j}$ $\qquad$ $B = (-4)\mathbf{i} + (-3)\mathbf{j}$

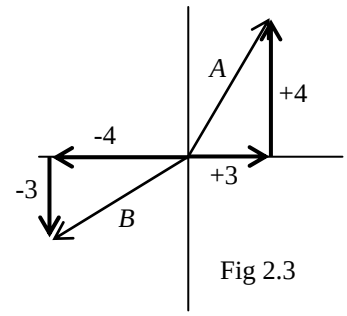


Fig 2.3

2. This becomes advantageous if vectors A and B need to be added.

   Find the resultant of the x vectors: $\boxed{R_x = A_x + B_x} = (+3) + (-4) = -1$

   Find the resultant of the y vectors: $\boxed{R_y = A_y + B_y} = (+4) + (-3) = +1$

   Then combine them to find R: $\boxed{R = \sqrt{R_x^{\,2} + R_y^{\,2}}} = \sqrt{(-1)^2 + (+1)^2} = 1.41$

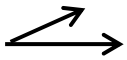
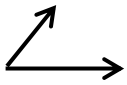
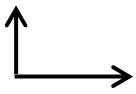
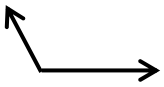
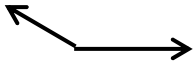
   Find the direction: $\boxed{\theta = \tan^{-1}\frac{R_y}{R_x}} = \tan^{-1}\frac{(+1)}{(-1)} = -45^o$, but this a 2nd quadrant angle and must be adjusted: $180^o - 45^o = 135^o$. The final vector is has a magnitude of 1.41 and a direction of 135°.

3. Finding the components simplify problems throughout physics. In Newtonian Mechanics motion, force, and momentum often act at an angle to the *x*- or *y*-axis. Fortunately these vector quantities can be resolved into component vectors along the *x*- or *y*-axis. In addition, equations for motion, force, and momentum can be calculated in the *x*-direction independent of what is happening in the *y*-direction, and vice versa. The first example of this will be projectile motion. In projectile motion there will be a variety of initial and final of velocities at angles.

Scalar (Dot) Product of Two Vectors: The dot product (**A**·**B**) of two vectors **A** and **B** is a scalar and is equal to $AB\cos\theta$. This quantity shows how two vectors interact depending on how close to parallel the two vectors are. The magnitude of this scalar is largest when $\theta = 0^o$ (parallel) and when $\theta = 180^o$ (anti-parallel). The scalar is zero when $\theta = 90^o$ (perpendicular).

Vector (Cross) Product: The cross product (**A**×**B**) of two vectors **A** and **B** is a third vector **C**. The magnitude of this vector is $C = AB\sin\theta$. The direction of this vector is determined by the right-hand-rule. The order that the vectors are multiplied is important (not commutative) If you change the order of multiplication you must change the sign, **A**×**B** = – **A**×**B**. The magnitude of this vector is largest when $\theta = 90^o$ (perpendicular). The vector is zero when $\theta = 0^o$ (parallel) or when $\theta = 180^o$ (anti-parallel).

Examples: In the following examples vector **A** = 3 and **B** = 2. The direction of vector **A** will vary.

Vectors A & B	Scalar (Dot) Product	Vector (Cross Product)
	$(3)(2)\cos 0^o = \boxed{6}$	$(3)(2)\sin 0^o = \boxed{0}$
	$(3)(2)\cos 30^o = \boxed{5.2}$	$(3)(2)\sin 30^o = \boxed{3.0}$
	$(3)(2)\cos 60^o = \boxed{3}$	$(3)(2)\sin 60^o = \boxed{5.2}$
	$(3)(2)\cos 90^o = \boxed{0}$	$(3)(2)\sin 90^o = \boxed{6}$
	$(3)(2)\cos 120^o = \boxed{-3}$	$(3)(2)\sin 120^o = \boxed{5.2}$
	$(3)(2)\cos 150^o = \boxed{-5.2}$	$(3)(2)\sin 150^o = \boxed{3}$
	$(3)(2)\cos 180^o = \boxed{-6}$	$(3)(2)\sin 180^o = \boxed{0}$

Another way: When cos (parallel) appears in a formula you need two vectors that are parallel (+) or anti-parallel (–). Just use your trig skills to find a component of one of the vectors that points in the same direction as the other vector. In the examples above, find the component of vector **B** that is parallel to vector **A**. When sin (perpendicular) appears in a formula, find the component of vector **B** that is perpendicular to vector **A**.

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