3–19 *RC* Circuits

Note: *RC* Circuits are needed for AP Physics C: physical science majors, calculus based.

***RC* Circuits:** Circuits containing both resistors and capacitors in series. The circuit in Fig 19.1 is an *RC* circuit, composed of *emf* source ε, resistor R, capacitor C, and switch S. The circuit in Fig 19.2 is a resistor circuit when the switch is in one position and a capacitor circuit when the switch is in the other position. Do not start the math without looking closely at the diagrams. We will focus on the *RC* circuit in Fig 19.1. You already have the skills to solve the other circuit.

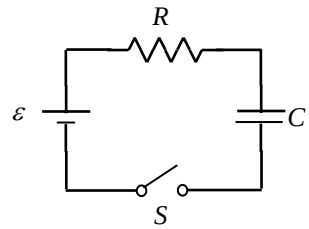

Fig 19.1

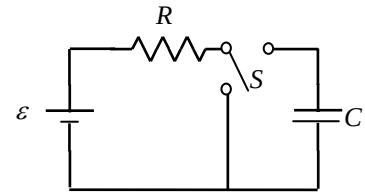

Fig 19.2

Initial conditions: Before the switch is thrown, in Fig 19.1, there is no current in the circuit. The battery produces an *emf*, but the circuit is incomplete. There is no electric field in the circuit and the charges lack a loop to follow. There are no charges built up on the capacitor. There is no resistance in the resistor.

Instantaneous Conditions, Right When Switch is Thrown (time t_0): An electric field traveling at the speed of light extents from the positive terminal of the battery to the negative terminal. The electric field accelerates charges (electrons moving opposite the field, from the negative terminal to the positive terminal. They do not accelerate to the speed of light as they encounter resistance (much like a sky diver hits terminal velocity due to air resistance). Initially there are no charges on the capacitor. This means there are no charges repelling the first charges arriving at the capacitor. So, the full current of these speeding charges move through the resistor. Initially the circuit behaves as though the capacitor does not exit, as shown in Fig 19.3. At this point the formula $V = IR$ applies, if it is adapted to show *emf* and initial current I_0 (at time t_0), as follows, $\varepsilon = I_0 R$. Then rearrange to solve for the initial current. $\boxed{I_0 = \frac{\varepsilon}{R}}$

A more formal approach uses Kirchhoff's 2nd Rule: Potential around a closed loop adds to zero. The potential of the battery minus the resistor and minus the capacitor must equal zero., $V - IR - \frac{Q}{C} = 0$. At this time there is no charge stored on the capacitor, so $Q = 0$. Substituting $\varepsilon - I_0 R - \left(\frac{0}{C}\right) = 0$, and solving $\boxed{I_0 = \frac{\varepsilon}{R}}$ leads to the same answer.

Right now the potential Difference of the battery appears entirely across the resistor.

At some time t later: Once charge begins to flow as a current, it is deposited on the capacitor. The charges are loaded onto the capacitor very easily at first. But, as the charges build up so does the electrostatic repulsion. This process is a logarithmic process. It begins quickly, with a lot of charge being deposited. Then it becomes more and more difficult to fight the electrostatic repulsion and the charging gradually tapers off.

The current in the circuit at some time t is solved using $\boxed{I_t = \frac{\varepsilon}{R} e^{-t/RC}}$. The value RC has special significance. It is called the time constant τ, where $\boxed{\tau = RC}$. It is the time for the current to decrease to $1/e$ of its initial value.

Right now the potential Difference of the battery is split between the resistor and the capacitor.

After a very long time: When a long time has passed the capacitor will become full. As a result no current will flow in the circuit. No current is flowing through the resistor, so it is as thought the resistor does not exist. $C = \frac{Q}{V}$. Rearrange to solve for potential $\boxed{V = \frac{Q}{C}}$. Using Kirchhoff's 2nd Law would give the same result. $V - IR - \frac{Q}{C} = 0$ becomes $\varepsilon - (0)R - \frac{Q}{C} = 0$ which simplifies to $\boxed{V = \frac{Q}{C}}$

Right now the potential Difference of the battery appears entirely across the capacitor.

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Capacitor Charge vs. Time Graph: Fig 19.3 shows the charge on the capacitor over time. As discussed before the charge on the capacitor is zero initially. When the current begins to flow the charges initially load onto the plates of the capacitor very quickly. In the beginning there are few charges on the plates that would repel additional charges. But, as the capacitor become more full the electric potential has to push charges against an ever increasing electrostatic repulsion. It becomes harder and harder to load more charge. Eventually the potential difference of the capacitor equals the potential difference of the battery. No more charges can load at this point. The maximum charge on the capacitor at this point is $\boxed{Q = C\varepsilon}$. It is shown as a dashed horizontal line marking the limit of charge on the graph. When the time constant is reached the capacitor is 63.2% charged.

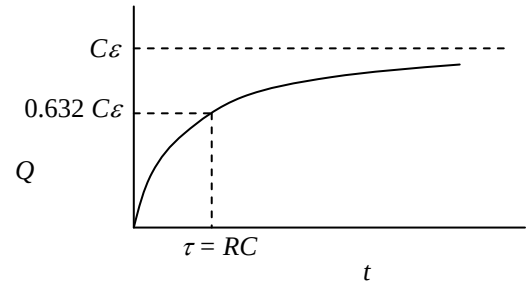

Fig 19.3

Current vs. Time Graph: Fig 19.4 shows the current in the circuit over time. Initially the current flows as though the capacitor is not present, $\boxed{I_0 = \frac{\varepsilon}{R}}$. But, as the capacitor charges the current decreases over time, according to $\boxed{I_t = \frac{\varepsilon}{R} e^{-t/RC}}$. Eventually when the capacitor reached a full charge the current stops entirely. $\boxed{I_0 = 0}$ charge on the graph. When the time constant is reached the current is at 36.8% of its initial value.

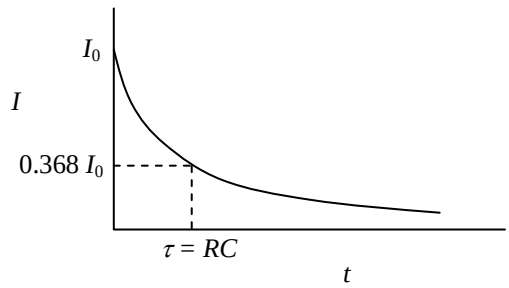

Fig 19.4

Discharging a Capacitor: The battery is removed from the circuit, Fig 19.5. The switch is thrown and the charge stored on one of the plates moves through the circuit and the resistor, toward the other plate. This process continues until the potential difference, originally created by the capacitor, drops to zero. Current flows very quickly at first, and then tapers to zero.

The charge on the capacitor varies by $\boxed{Q_t = Qe^{-t/RC}}$

The current in the circuit varies by $\boxed{I_t = -\frac{Q}{RC} e^{-t/RC}}$ and $\frac{Q}{RC} = I_0$

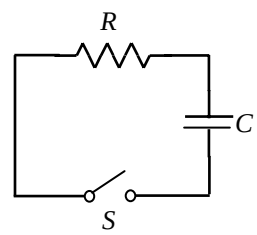

Fig 19.5

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