3–22 Faraday's Law of Induction

Electromagnetic Induction (Right Hand Rule, Case 4)

If a current carrying wire in a larger magnetic field can be made to jump, then could the reverse be true. Could you force a wire to jump into a large magnetic field and then would this in turn cause a current to flow in the wire. This is the basis of electricity production, and is how an electrical generator operates. We must introduce a new quantity known as flux.

Magnetic Flux: $\boxed{\phi_m = B \cdot Area}$ The strength of a magnetic field moving through an area of space, such as a loop of wire. There is an amount of gravity flux through the surface of the earth. There is electrical flux through the space between the plates of a capacitor, and around charged other charged objects. There is magnetic flux around any magnet. In simple terms the flux can be thought of as the amount of field lines passing through a defined area of space. While the field lines aren't really there they are a way of viewing the invisible field. The more lines drawn, and the closer they are to one another the stronger the field and the greater the flux. We will consider the flux through a single simple loop of wire that is located in a magnetic field, Fig.19.9. The wire is full of tiny little magnets (charges). If the wire is forced to move by us (nuclear power, hydroelectric, etc.) these tiny magnets are forced to move in the field. The force of magnetism from the larger surrounding magnetic field, created by a large fixed magnet, will put a force on the tiny charges. As the wire moves the charges move due to the force magnetic, causing a current to flow in the wire. So a stationary loop generates no electricity. Therefore you need a moving loop and as it turns out you need a ***changing flux***. You can change flux several ways: You can hold the magnet stationary and move the wire changing the area (into the field or out of the field). You can rotate the wire in the field, also changing the area. You could hold the wire stationary, and move the magnet, changing the field B's strength as distance changes. You could generate the magnetic field with another current carrying wire, and by changing the current, you also change the field strength.

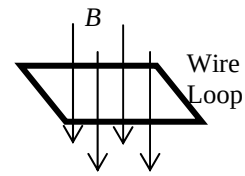

Fig 19.9

Faraday's Law of Induction: $\boxed{\varepsilon_{avg} = -\frac{\Delta\phi_m}{\Delta t}}$ Changing flux means the charges in the wire experience a changing magnetic field. An electromotive force is induced. This $\boldsymbol{\varepsilon}$ causes the charges in the wire to move. The charges are trapped in the wire loop and can only travel in a direction consist with both the force and the confines of the wire itself.

Calculus: Faraday's Law is formally written as $\boxed{\varepsilon_{avg} = -\frac{d\phi_m}{dt}}$. If the circuit is a coil consisting of N loops then the total induction is given by $\varepsilon_{avg} = -N\frac{d\phi_m}{dt}$. In algebraic terms this becomes $\varepsilon_{avg} = -N\frac{\Delta\phi_m}{\Delta t}$

Electromotive Force, *emf*: Induction creates electromotive force *emf* or ε, which really isn't a force, rather it is electric pressure or voltage. However, every device that generates electricity has a resistance, known as ***internal resistance***. So *emf* is actually the amount of voltage that you could produce if there were no internal resistance in a battery, power supply, generator, or wire loop used to move a current. If internal resistance is included then the equation for induced voltage is $\boxed{V = \varepsilon - Ir}$. Use you're rules for series circuits to deal with the small internal resistance. Treat the internal resistance as a resistor that happens to be next to the battery or power supply. ***In simple problems the internal resistance will be negligible*** $\boxed{Ir = 0}$, similar to a frictionless/airless mechanics problem. Then $\boxed{V = \varepsilon}$.

Lenz's Law: A magnetic field can move charges in a wire by electromagnetic induction. Once the charges start moving they generate their own magnetic field. This new field attempts to counteract the original magnetic field, and is opposite in direction. In order to have opposite direction the charged particles must move in a direction opposite the right hand rule. So in electromagnetic induction the ***minus sign*** in the above equation is a ***reminder that the charges are moving opposite in direction***. **Use the right hand rule as before, but reverse the direction of current.**

Example 22.1: Electromagnetic Induction A solenoid with N turns, length ℓ, and current I is surrounded by a ring of conducting material. **What is the induced *emf* in the ring?** Combine $\boxed{\varepsilon_{avg} = -N\frac{d\phi_m}{dt}}$, $\boxed{\phi_m = \int \mathbf{B} \cdot d\mathbf{A}}$, $\boxed{B_S = \mu_0 nI}$, and $\boxed{n = \frac{N}{\ell}}$ to produce $\boxed{\varepsilon_{avg} = N\frac{\mu_0 \frac{N}{\ell} IA}{t}}$. Simplfy to get $\boxed{\varepsilon_{avg} = \frac{\mu_0 N^2 IA}{\ell t}}$

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Induction by Moving a Loop Into or Out of a Magnetic Field

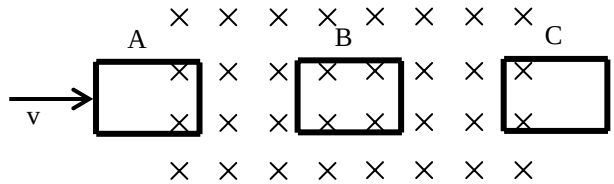


Fig 19.10

The loop, in Fig 19.10, is forced (palm pushing represents the direction the wire is forced) toward the right, with velocity v. Look for the part of the loop that is cutting through field lines. This will induce the voltage necessary to move the current. The front edge of the loop is cutting through x's and thus induces a upward current in the leading edge of the loop at location A. The upward motion of charge in the front of the loop causes the rest of the charges in the loop to move as well. The whole motion of charge, induced current is counter clockwise in the entire loop (remember Lenz's Law and the reversal of current direction in induction).

The induced voltage is $\varepsilon_{avg} = \dfrac{\Delta\phi_m}{\Delta t}$ $\qquad \phi = B \cdot Area$ $\qquad \varepsilon_{avg} = \dfrac{B \cdot \Delta Area}{\Delta t}$

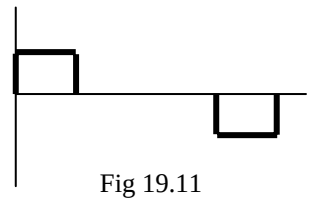


Fig 19.11

The area of the loop in the field is growing, as the loop enters the field, and thus the flux is growing as well. But, the amount of area changing every second is the same since the loop is moving at constant velocity. The front edge of the loop cuts through the same number of field lines every second. ***emf*** does not depend on the size of the area (flux does), rather it depends on the rate of change in flux. This is constant, due to the constant velocity of the loop. Once the entire loop enters the field, position B, a counterclockwise current is induced in the front edge of the loop, while a clockwise current is induced by the back edge. The two opposite currents cancel each other. When the front edge, position C, has left the field, the back of the loop creates a clockwise current. The field is assumed to end at the edge of the x's, or dots for the field drawn out of the page (+z). It is assumed to be a uniform field inside the x's. Graphically the induced emf is shown to the right.

Metal Bar Sliding Across a Loop of Wire

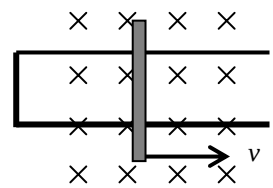


Fig 19.12

If you have a stationary loop of wire placed in a magnetic field, and a metal bar is dragged across the wire loop (Fig 19.12) an electromotive force is produced $\boxed{\varepsilon = B\ell v}$. ***B*** is the magnetic field, ℓ is the length of the bar, and v is the velocity of the bar. This is derived using the formulas above.

$$\varepsilon_{avg} = \frac{\Delta\phi_m}{\Delta t} \qquad \phi = B \cdot Area \qquad \varepsilon_{avg} = \frac{B \cdot \Delta Area}{\Delta t} \qquad \varepsilon_{avg} = \frac{B \cdot \Delta(\ell \times d)}{\Delta t} \qquad \varepsilon_{avg} = B \cdot \ell \frac{\Delta d}{\Delta t}$$

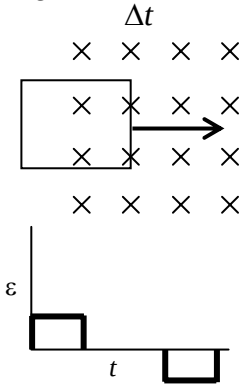
$$\boxed{\varepsilon = B\ell v}$$

Graphing Induced Electromagnetic Force $\boxed{\varepsilon = -\dfrac{\Delta\phi_m}{\Delta t}}$

Remember, current can only be generated by a changing flux. So a closed loop of wire must move through the field, or the loop must be getting larger, or the loop must be rotating, or the magnetic field must be changing.

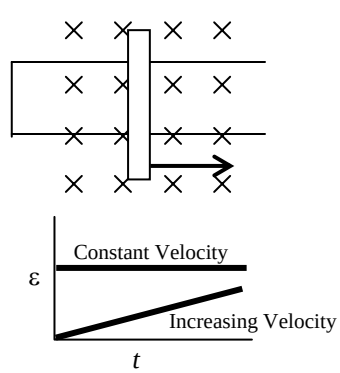
Loop moved thru

$$\varepsilon = \frac{B \cdot \Delta(A)}{\Delta t}$$



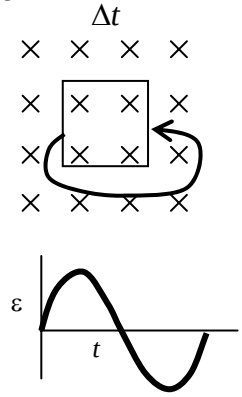
Bar moved, enlarging loop

$$\varepsilon = B\ell v$$



Loop rotates

$$\varepsilon = \frac{B \cdot \Delta(A)}{\Delta t}$$



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