Unit 2 Fluids and Thermal Physics

2–12 Fluid Mechanics

Solids: ***Condensed matter*** with definite volume and shape. High intermolecular forces create rigid structure. Force can deform it.

Liquids: ***Condensed matter and fluid*** with fairly definite volume, but takes the shape of its container. Loose intermolecular forces.

Gases: ***Fluid only***. Easily compressed, taking shape & volume of its container. Particle speed interferes with intermolecular force.

Fluids: ***Substance that can flow*** (gases and Liquids). Outside forces cause, unconfined, fluids to flow.

Pressure: ***Force applied over an area***. $\boxed{p = \frac{F}{A}}$ F is perpendicular to the surface (normal). Pressure is a scalar quantity.

Measured in Pascals. 1 Pa = 1 N/m^2. 1 atm = 101325 Pa, which simplifies to $\boxed{1atm = 1\times10^5 Pa}$

Density: ***The amount of matter occupying a certain amount of space***. $\boxed{\rho = \frac{m}{V}}$ Measured in kg/m^3 (in chem. g/cm^3)

The density of water is easy to memorize. Water is the most common substance on earth, so chemists set its value at 1 g/cm^3.

Water is the reference standard by which all other densities are compared. But, physics operates on a larger scale with different units. $\rho_{H_2O} = 1.00\, g/cm^3 = \boxed{1000\, kg/m^3}$

Pressure and Depth: ***Pressure increases with depth.*** A column of fluid (air we breath or water in oceans) has weight and puts a force on the fluid below it, Fig 12.1. The deeper one is (on the surface of earth for air, or at the bottom of the ocean for water) the greater the force and pressure. The equation for pressure at depth is derived by combining the following four equations.

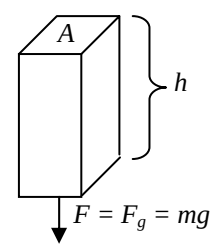

Fig 12.1

$$p = \frac{F}{A}, \quad F_{due\ to\ fluid\ above} = mg, \quad \rho = \frac{m}{V}, \text{ and } \quad V = hA \quad \longrightarrow \quad p = \rho g h$$

which becomes $\boxed{p = p_0 + \rho g h}$ p_0 is the pressure on the surface. This could be caused by a piston in a closed tube, or by the atmospheric pressure for a fluid whose surface is exposed. Example: a scuba diver, at a depth of 80 ft in saltwater, experiences a pressure of

$p = p_0 + \rho g h = \left(1\times10^5\right) + \left(1.03\times10^3\right)\left(9.8\right)\left(24.4\right) = \boxed{3.46\times10^5 Pa} = 3.46 atm$ (3.5 × atmospheric pressure at sea level). How does this translate to force? Calculate the force on 1 square meter $p = F/A$, so

$F = pA = \left(3.46\times10^5 Pa\right)\left(1m^2\right) = \boxed{34600 N}$

Pascal's Principle: ***Pressure applied to an enclosed fluid is transmitted, undiminished, to every point in the fluid and to the walls of the container.*** This explains the workings of hydraulic lifts, jacks, and brakes. The pressure put in (input) at one end is equal to the pressure put out (output) at the other.

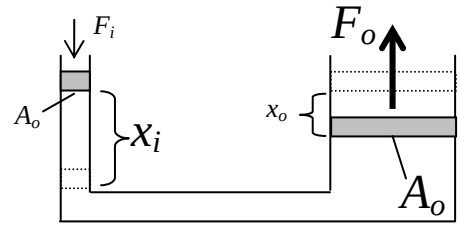

Fig 12.2

$p_{input} = p_{output}$. Substitute $p = \frac{F}{A}$ on each side to get $\boxed{\frac{F_i}{A_i} = \frac{F_o}{A_o}}$.

A small input force on a small area creates a large output force on a large area. Work done pushing the piston can also be analyzed.

$W_i = W_o$ which is $F_i x_i = F_o x_o$ and becomes $\boxed{F_o = F_i \frac{x_i}{x_o}}$

The input distance must be large to get a large output force, F_o.

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Buoyancy: Things float because they are buoyed upward, which requires an upward net force on the object. It is caused by the difference in pressure $\left(\Delta p = p_{bottom} - p_{top}\right)$ between the bottom of the object and the top of the object.

$p = \frac{F}{A}$ combined with $\Delta p = \rho g h_{bottom} - \rho g h_{top} = \rho g\left(h_{bottom} - h_{top}\right)$ results in $\frac{F}{A} = \rho g\left(h_{bottom} - h_{top}\right)$ which is

$F = \rho g\left(h_{bottom} - h_{top}\right)A$ or $\boxed{F_{buoy} = \rho V g}$ Remember this is the ***density of the fluid*** and the ***volume of the fluid displaced***. This is the equation for the weight of the fluid displaced. $F_{buoy} = \rho V g = \frac{m}{V} V g = \boxed{mg}$ of the fluid.

Archimedes' Principle: ***<u>A body immersed wholly or partially in a fluid is buoyed up by a force equal in magnitude to the weight of the volume of fluid it displaces.</u>***

Buoyancy and Density: Density tells whether it will sink or float. Fig 12.3 diagrams several masses in a fluid.

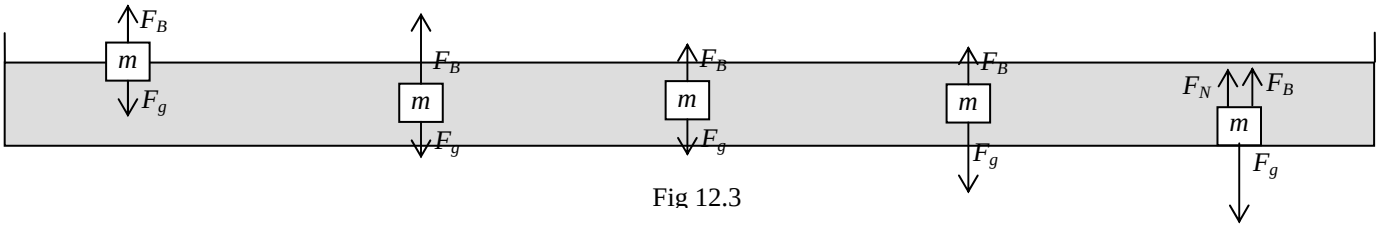

Fig 12.3

Floats out of water	Rising to Surface	Neutral Buoyancy	Sinking to Bottom	Resting on Bottom
$F_B = F_g$	$F_B > F_g$	$F_B = F_g$	$F_B < F_g$	$F_B + F_N = F_g$
$\rho_{obj} < \rho_{Fluid}$	$\rho_{obj} < \rho_{Fluid}$	$\rho_{obj} = \rho_{Fluid}$	$\rho_{obj} > \rho_{Fluid}$	$\rho_{obj} > \rho_{Fluid}$
$m_{obj} = m_{Fluid\ Displaced}$	$m_{obj} < m_{Fluid\ Displaced}$	$m_{obj} = m_{Fluid\ Displaced}$	$m_{obj} > m_{Fluid\ Displaced}$	$m_{obj} > m_{Fluid\ Displaced}$
$V_{obj} > V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$	$V_{obj} = V_{Fluid\ Displaced}$

Specific Gravity: ratio of the density of a substance (ρ_s) to the density of water (ρ_w). $\boxed{sp.gr. = \rho_s / \rho_w}$

Example 1-1: Buoyancy

A 2.0 kg cube, that is 10.0 cm in length on each side, is suspended by both a gray fluid and by a spring as shown in Fig 12.4a. The spring constant is 100 N/m and the spring is stretched 0.15 m.

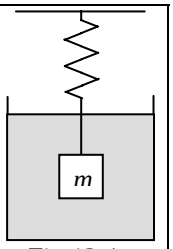

Fig 12.4a

What is the density of the fluid? This is a balanced force problem. The object is not moving. Use the FBD in Fig 12.4b to sum the forces.

$\sum F = F_S + F_B - F_g \qquad \sum F = 0$

$F_S + F_B = F_g$ Upward force spring and force buoyancy equals the downward force gravity.

$kx + \rho V g = mg$

$\rho = \frac{mg - kx}{Vg} \qquad \rho = \frac{(2.0)(9.8) - (100)(0.15)}{(0.1)^3(9.8)} = \boxed{469\frac{kg}{m^3}}$

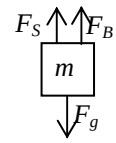

Fig 12.4b

Fluid Dynamics: The overall flow of a fluid can be described, but the motion of the individual molecules or atoms that compose the fluid cannot. We use a theoretical model, known as an ***<u>ideal fluid</u>***. Ideal fluids have four characteristics.

Steady Flow: ***<u>Smooth flow</u>***, where all the particles of a fluid have the same velocity as they pass a given point. The particle path is depicted by streamlines that never cross. This requires low velocities, since high flow rate results in eddies, particularly near boundaries, which give rise to turbulence. Relative velocity is shown by drawing faster flowing fluid streamlines closer together.

Irrotational Flow: No net angular velocities in any portion of the fluid. i.e., there is no possibility for eddies and turbulence.

Nonviscous Flow: A nonviscous fluid flows freely without energy loss. Viscosity is a fluid's internal friction, and is neglected.

Incompressible Flow: Fluid's density is constant. Gases are not incompressible, but sometimes they approximate incompressibility.

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Equation of Continuity: ***<u>If there is no loss of fluid in a closed tube the mass entering must equal the mass leaving the tube.</u>*** In Fig 12.5 a fluid flows through a tube that is narrow at first and then widens. The volumes of the narrow and wide portions are:

$$V_1 = A_1 \Delta x_1 \qquad V_2 = A_2 \Delta x_2$$

Substitute $\Delta x = v\Delta t$: $\quad V_1 = A_1 v_1 \Delta t \qquad V_2 = A_2 v_2 \Delta t$.

Volume relates to mass.

$\rho = \dfrac{m}{V}$ so $m = \rho V$

Substituting $\Delta m = \rho \Delta V$: $\quad \Delta m_1 = \rho_1 A_1 v_1 \Delta t \qquad \Delta m_2 = \rho_2 A_2 v_2 \Delta t$

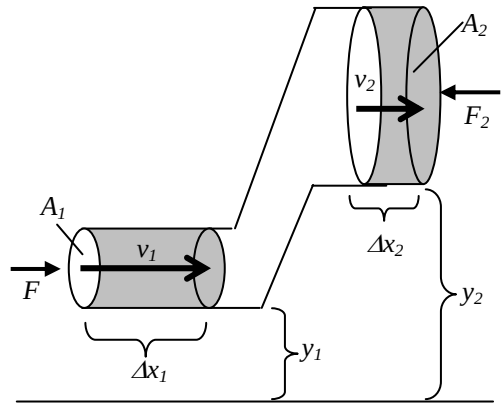

Fig 12.5

But, the two shaded sections of pipe were picked because they have equal volumes, one long and thin, the other short and wide. So if the volumes are the same, then the mass of fluid in the first section equals the mass of fluid in the second section.

$\Delta m_1 = \Delta m_2$ and thus $\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$. Time cancels, as does density. In ideal fluids density is assumed to be uniform.

Flow Rate Equation: $\boxed{A_1 v_1 = A_2 v_2}$ ***<u>Flow velocity is greater where the cross sectional area is smaller.</u>*** $v_2 = \dfrac{A_1}{A_2} v_1$

Now Analyze the Work done by external forces at ends of the tube in the figure above right.

F_1 is positive (matches motion), while F_2 is negative (opposite motion). $\qquad W = F_1 \Delta x_1 - F_2 \Delta x_2$

Substitute $p = \dfrac{F}{A}$ written as $F = pA$, along with $\Delta x = v\Delta t$ $\qquad W = p_1 A_1 v_1 \Delta t - p_2 A_2 v_2 \Delta t$

Using the Flow Rate Equation just derived, $A_1 v_1 = A_2 v_2$ and we simplify to $\qquad W = A_1 v_1 \Delta t \left(p_1 - p_2 \right)$

Then use the mass relationship from above, $\Delta m_1 = \rho_1 A_1 v_1 \Delta t$ $\qquad W = \dfrac{\Delta m}{\rho}\left(p_1 - p_2 \right)$

Work is a change in energy $W = \Delta K + \Delta U$ $\qquad \dfrac{\Delta m}{\rho}\left(p_1 - p_2 \right) = \Delta K + \Delta U$

Substitute the formulas for ΔK and ΔU

$$\frac{\Delta \cancel{m}}{\rho} p_1 - \frac{\Delta \cancel{m}}{\rho} p_2 = \frac{1}{2}\Delta \cancel{m} v_2^{\,2} - \frac{1}{2}\Delta \cancel{m} v_1^{\,2} + \Delta \cancel{m} g y_2 - \Delta \cancel{m} g y_1$$

Cancel mass and multiply all by density $\qquad p_1 - p_2 = \dfrac{1}{2}\rho v_2^{\,2} - \dfrac{1}{2}\rho v_1^{\,2} + \rho g y_2 - \rho g y_1$

Rearrange separating 1's and 2's $\qquad p_1 + \rho g y_1 + \dfrac{1}{2}\rho v_1^{\,2} = p_2 + \rho g y_2 + \dfrac{1}{2}\rho v_2^{\,2}$

Bernoulli's Equation: $\qquad \boxed{p + \rho g y + \dfrac{1}{2}\rho v^2 = constant}$

Applying Bernoulli's Equation to various situations.

- Horizontal flow $\left(y_1 = y_2 \right)$: $\boxed{p + \dfrac{1}{2}\rho v^2 = constant}$ Indicates that pressure decreases if fluids speed increases or vice versa.
- Looking at both Bernoulli's and the Continuity Equation: Reduce pipe cross section, then velocity goes up, and pressure down.
- Partially responsible for lift: Air over the curved surface of a wing goes faster, with lower pressure. So the slow high pressure air on the bottom of the wing pushes upward.
- Fluids at rest $\left(v_2 = v_1 = 0 \right)$: $\boxed{p_2 - p_1 = \rho g \left(y_1 - y_2 \right)}$ which is the pressure depth relationship.

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Example 1-2: Buoyancy

A tube, shown in Fig 12.6a, that has an opening at the top and a hole in the side is continuously filled with water. It is filled so that the water level remains constant, even though water is leaking from the hole in the side.

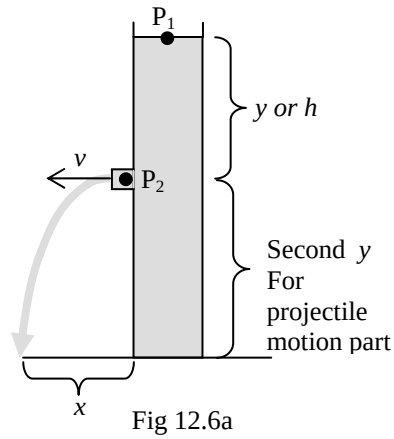

Fig 12.6a

What is the velocity of the water coming out of the hole in the side?

Use Bernoulli's Law $\boxed{p + \rho g y + \frac{1}{2}\rho v^2 = constant}$

Pick two convenient locations: The surface of the water (point 1) and the hole in the side (point 2). The fluid flows from the surface $\boxed{p_1 + \rho g y_1 + \frac{1}{2}\rho v^2{}_1 = constant}$ to the hole $p_2 + \rho g y_2 + \frac{1}{2}\rho v^2{}_2 = constant$.

It is the same fluid, so the constant at both points must be the same. Therefore,

$$p_1 + \rho g y_1 + \frac{1}{2}\rho v^2{}_1 = p_2 + \rho g y_2 + \frac{1}{2}\rho v^2{}_2$$

The tube is open to the atmosphere at both ends, so $p_1 = p_2$

$$\cancel{\rho} g y_1 + \frac{1}{2}\cancel{\rho} v^2{}_1 = \cancel{\rho} g y_2 + \frac{1}{2}\cancel{\rho} v^2{}_2$$

Since the level of the water is maintained at a constant point the velocity at point 1 is zero.

$g y_1 = g y_2 + \frac{1}{2} v^2{}_2$ Which simplifies to $v = \sqrt{2g\left(y_1 - y_2\right)}$ or $\boxed{v = \sqrt{2g\Delta y}}$

What if we look at this differently?

Suppose we follow a single water molecule from point 1 to point 2. Pretend it follows the path (in an energy problem path doesn't matter, so any path will give the same answer) diagramed in Fig. 12.6b. What would its velocity be? Height is turning into velocity. $\cancel{m} g h = \frac{1}{2}\cancel{m} v^2$

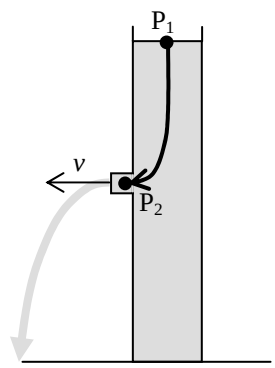

Fig 12.6b

$v = \sqrt{2gh}$ or $\boxed{v = \sqrt{2g\Delta h}}$. This looks familiar! Notice that mass cancelled, so it does not matter if it is one molecule or a stream of molecules. Funny how Physics ties together and different approaches to problems can yield the same answers.
Notice that the water leaving the opening is a horizontally launched projectile.

$\boxed{y = \frac{1}{2}gt^2}$ and $\boxed{x = v_{0x}t}$ Just be careful this y is different from the y used in Bernoulli's equation. Note the labels on the right side of Fig 12.6a

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